Measuring Characteristic Impedance

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Introduction

There are many instances where designing and constructing one's own transmission line is required. What comes to mind are feeders for log-periodic dipole arrays (LPDAs), all kinds of open-wire transmission line, stripline filters, microstrip circuits, twisted-pair lines, Guanella chokes and baluns, and even the lanes of a computer memory bus.

This site actually hosts transmission line calculators for:

- parallel circular conductor transmission line,
- parallel square conductor transmission line, and
- star quad transmission line.

When building transmission line yourself, it is of course important to check whether the characteristic impedance corresponds to the calculated value. This article explains how the characteristic impedance of a transmission line can easily be determined from two vector network analyser (VNA) measurements.

Procedure & formula

The characteristic impedance Z_c of a length ℓ of transmission line can be derived from measuring its input impedance Z_{in} once with the transmission line terminated in a short and a second time left open. Obviously, prior to connecting the transmission line, the VNA is calibrated at its device under test (DUT) port with a short, open and 50 Ω load (SOL).

It can be shown (see below), that the characteristic impedance Z_c —which is a complex number when there are losses— corresponds to:

$$Z_c = \sqrt{Z_{in,\,short} \cdot Z_{in,\,open}} \tag{1}$$

Measuring frequency

The measuring frequency of a given length ℓ of transmission line cannot be chosen arbitrary. In order to reduce measuring errors to a minimum, the measuring frequency needs to be such that the electrical length ℓ —which takes into account the velocity factor— corresponds more or less to an odd multiple of eights of a wavelength.[1,2] This is explained further on.

Measuring balanced line

The VNA must be capable of measuring balanced impedances if the line conductors are symmetrical (e.g. a parallel-wire line or a shielded pair).[2] This may require a balun installed in front of the calibration plane. However, a balun is not required when an asymmetrical (coaxial) VNA works off batteries without being connected to ground. This is also the case if a headless asymmetrical VNA is moreover connected wirelessly to its steering computer. The mini Radio Solutions miniVNA PRO is a great example of a wireless, battery operated VNA device.

Derivation

The input impedance $Z_{\mathrm{in,\,short}}$ of a transmission line stub terminated in a short circuit is given by:

$$Z_{\mathrm{in,\,short}} = Z_\mathrm{c} \tanh\left(\gamma\ell\right) \approx \mathrm{j} \tan\left(\beta\ell\right) Z_\mathrm{c} \tag{2}$$

where:
$\gamma = \alpha + \mathrm{j}\beta$ is the propagation constant γ,
α is the attenuation constant, and
β is the phase constant.

Whereas the input impedance $Z_{\mathrm{in,\,open}}$ of a transmission line stub terminated in an open circuit is given by:

$$Z_{\mathrm{in,\,open}} = Z_\mathrm{c} \coth\left(\gamma\ell\right) \approx -\mathrm{j} \cot\left(\beta\ell\right) Z_\mathrm{c} \tag{3}$$

Hence,

$$\sqrt{Z_{\mathrm{in,\,short}} \cdot Z_{\mathrm{in,\,open}}} = \sqrt{-\mathrm{j}^2 Z_\mathrm{c}^2} = Z_\mathrm{c} \tag{4}$$

However, if the transmission line would be almost an odd number of quarter wavelengths long, the angle $(\beta\ell)$ would be nearly an odd integer times $\pi/2$ radians. Then, $Z_{\text{in, short}}$ would approach an open circuit and $Z_{\text{in, open}}$ would approach a short circuit. This would render making accurate measurements extremely difficult.[1,3]

Conversely, if the transmission line would be almost an even number of quarter wavelengths long, then $Z_{\text{in, short}}$ would be very low and $Z_{\text{in, open}}$ would be extremely high.

Impedance measuring errors can be significantly reduced when the magnitudes of $Z_{\text{in, short}}$ and $Z_{\text{in, open}}$ are about the same and appropriate to the VNA. This happens when:

$$\tan(\beta\ell) \approx \cot(\beta\ell) \approx 1 \tag{5}$$

Hence,

$$\beta\ell \approx (2n+1)\frac{\pi}{4} \quad \Rightarrow \quad \ell \approx (2n+1)\frac{\pi}{4}\frac{\lambda}{2\pi} = (2n+1)\frac{\lambda}{8} \tag{6}$$

where:
n is an integer, and
$\beta \equiv \frac{2\pi}{\lambda}$ is the phase constant in $\frac{rad}{m}$.

In other words, the *electrical* length of the transmission line under test should measure more or less an odd number of eighth wavelengths.

However, if the overall attenuation is high, the variation of impedance with length is not so violent, and it might not be necessary to select the transmission line length with such care.[1,3]

References

1. Walter C. Johnson. *Transmission Lines and Networks*. McGraw-Hill Book Company; 1963.

2. Robert A. Chipman. *Theory and Problems of Transmission Lines*. McGraw-Hill Book Company; 1968.

3. Chemandy Electronics. Measuring characteristic impedance of PCB tracks using a vector network analyser. Published 2019. https://chemandy.com/technical-articles/measuring-track-characteristic-impedance/measuring-track-characteristic-impedance-article1.htm

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