3 Plane Surface Waves along Plane Layers of Isotropic Media

3.1 Definition

A plane surface wave is defined as a plane wave that propagates along a plane interface of two different media without radiation [1, p.5].
Note that radiation in this context is construed as being energy converted from the surface wave field to some other field form.

Plane surface waves are inhomogeneous waves because the field is not constant along surfaces of constant phase. In fact, in the case of a surface wave the field decays exponentially over the wavefront with increase of distance from the surface.

There are E-type and H-type surface waves. The field of an E-type plane surface wave is depicted in Figure 3.1. For an H-type wave the E- and H-fields are interchanged and one of the fields is reversed in sign. Explicit equations for the fields of plane surface waves along various structures will be derived rigorously later in this chapter.

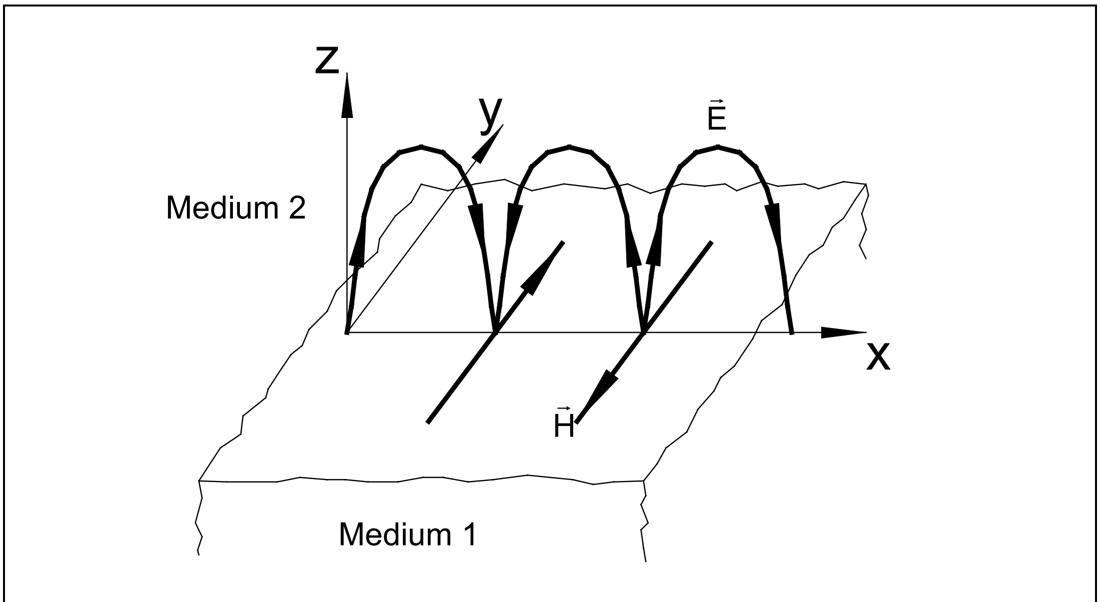


Figure 3.1: The field of an E-type plane surface wave

3.2 Plane Surface Waves and the Brewster Angle Phenomenon

There are many ways to explain the mechanism of surface waves. In the mid-fifties Barlow et al. introduced the concept of surface impedance for this purpose [1, pp. 15-17], [2], which will be explained in a later section. Earlier work by Zenneck (1907) associated plane surface waves with the Brewster angle phenomenon [1, pp. 29-33], [3, p. 697-701]. For this reason plane surface waves are sometimes also called *Zenneck waves*. Due to the many prevailing misconceptions, the relation between plane surface waves and the Brewster angle will receive some further attention here.

The Brewster angle is the angle of incidence at which a plane wave incident on a plane material interface is totally transmitted (i.e. without reflection) from one medium, called medium 2 here, into another medium, called medium 1. Both media are assumed to be half spaces. In lossless media, the Brewster angle phenomenon only occurs for perpendicular and parallel polarized incident plane waves. (The terms “perpendicular” and “parallel” refer to the orientation of the electric field intensity vector $\vec{E}_i$ of the incident plane wave with respect to the plane of incidence.) The Brewster angle is different for the two types of polarization. From Fresnel’s equations [4], it can be shown that the Brewster angle for perpendicular polarized incident plane waves is

$$\theta_{B\perp} = \operatorname{asin}\sqrt{\frac{1-\dfrac{\varepsilon_1\mu_2}{\varepsilon_2\mu_1}}{1-\left(\dfrac{\mu_2}{\mu_1}\right)^2}} \quad \text{and}$$

$$\theta_{B//} = \operatorname{asin}\sqrt{\frac{1-\dfrac{\varepsilon_2\mu_1}{\varepsilon_1\mu_2}}{1-\left(\dfrac{\varepsilon_2}{\varepsilon_1}\right)^2}}$$

for parallel polarized incident plane waves.

Above equations result in a complex value for the Brewster angle:

- if, for perpendicular polarization, $\mu_2 > \mu_1$,
- or if, for parallel polarization, $\varepsilon_2 > \varepsilon_1$,
- or if at least one of the two media has losses.

The physical meaning of *a complex angle of incidence* is an inhomogeneous plane wave (which in fact very much resembles a surface wave) incident at that angle [1, p. 30], [3, p. 717], [5]. When the Brewster angle is complex, the angle for which the magnitude of the reflection coefficient is a minimum, is called *the pseudo-Brewster angle* [1, p. 31]. The Brewster angle is also sometimes called *the polarizing angle* since a wave with both perpendicular and parallel components and which is incident at the Brewster angle will produce a reflected wave with only a perpendicular or parallel component [6, p. 617]. To summarize, *the only connection between surface waves and the Brewster angle lies in the fact that the inhomogeneous wave required by a complex Brewster angle resembles a surface wave.*

3.3 Plane Surface Waves, Total Reflection and Leaky Waves

An idea of what the different fields along a material interface may look like, can be obtained by employing ray-optics theory once again. The existence of propagating plane surface waves along a coated perfectly conducting plane, for example, can be explained with the help of the total reflection phenomenon as shown in Figure 3.2.

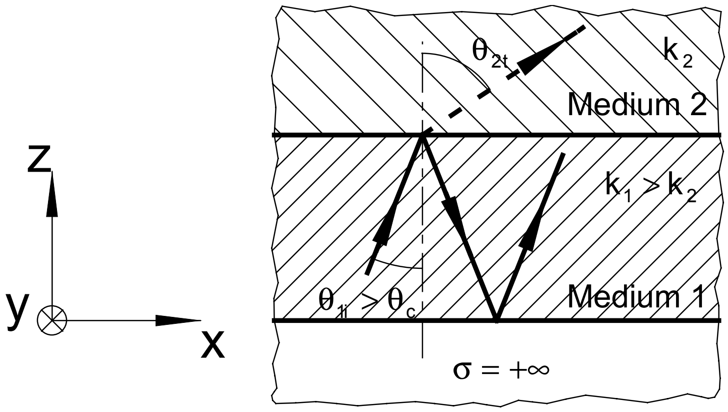


Figure 3.2: A ray-optics explanation for the propagation of plane surface waves along a coated perfectly conducting plane

Total reflection only occurs when a wave from medium 1 impinges upon medium 2 at an angle of incidence θ_{1i} equal to or exceeding *the critical angle* θ_c and then only if medium 1 is more "dense" than medium 2 (k_1>k_2).

An expression for the critical angle as a function of k_1 and k_2 can be obtained as follows. The relation between the angles of reflection and refraction is given by Snell's law $k_2 \cdot \sin(\theta_{2t}) = k_1 \cdot \sin(\theta_{1i})$

$$\Rightarrow \ \sin(\theta_{2t}) = \frac{k_1}{k_2}\sin(\theta_{1i}).$$

There will be no refracted wave if $\sin(\theta_{2t})$ is greater than one or equivalently, if $\sin(\theta_{1i}) > \frac{k_2}{k_1} = \sin(\theta_c)$.

Hence, $\theta_c = a\sin\left(\frac{k_2}{k_1}\right)$.

As suggested in Figure 3.2, total reflection can be accompanied by a surface wave propagating in medium 1, parallel with the material interface. What is not shown, is the fact that a surface wave field most of the times also extends into medium 2. This will be proven in the next section. Note that a surface wave can exist along a coated perfectly conducting plane only if the coating (medium 1) is more dense than the upper half space (medium 2) ($k_1 > k_2$). If this is not the case or if the angle of incidence θ_{1i} is smaller than the critical angle θ_c, part of the wave in medium 1 will be transmitted into medium 2 with each partial reflection. Therefore, the field will quickly attenuate in the x-direction. The resulting inhomogeneous plane wave is called *a leaky wave* and propagates away from the interface (Fig. 3.3b).

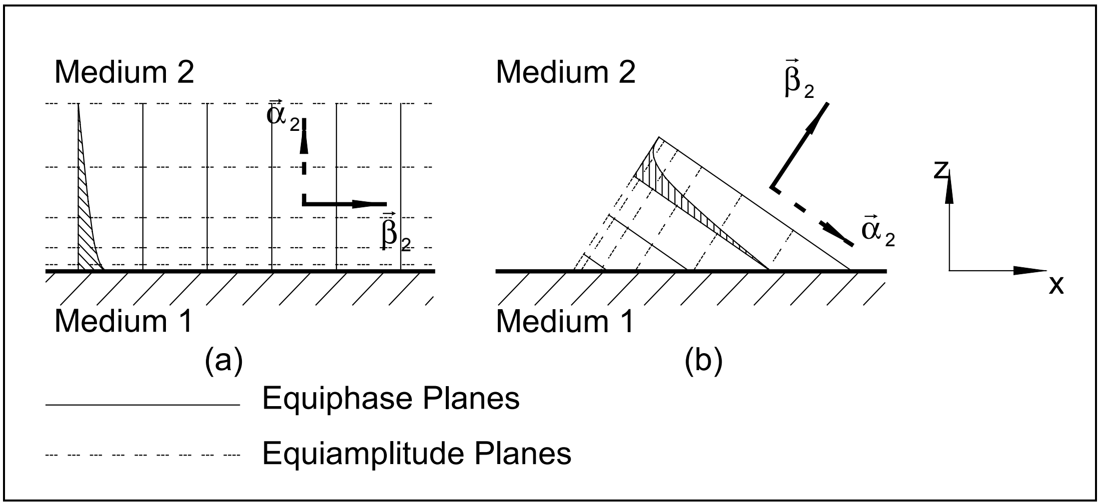


Figure 3.3: (a) A surface wave, (b) a leaky wave (Medium 2 is assumed to be loss free in this figure.)

Note that $\vec{\alpha}_2$ is perpendicular to $\vec{\beta}_2$ only if medium 2 is loss free ($\mathrm{Im}(k_2) = 0$). This will be shown now.

$k_2^2 = k_{2x}^2 + k_{2z}^2 = \left(\beta_{2x} - j\alpha_{2x}\right)^2 + \left(\beta_{2z} - j\alpha_{2z}\right)^2$

$\Rightarrow\ k_2^2 = \beta_{2x}^2 + \beta_{2z}^2 - \left(\alpha_{2x}^2 + \alpha_{2z}^2\right) - 2j\left(\alpha_{2x}\beta_{2x} + \alpha_{2z}\beta_{2z}\right).$

Only and only if $\mathrm{Im}(k_2) = 0\ \Rightarrow\ \alpha_{2x}\beta_{2x} + \alpha_{2z}\beta_{2z} = 0$

$\Rightarrow\ \vec{\alpha}_2 \cdot \vec{\beta}_2 = 0\ \Rightarrow\ \vec{\alpha}_2 \perp \vec{\beta}_2.$

For the sake of simplicity, $\vec{\alpha}_2$ will always be drawn for the case where medium 2 is loss free, i.e. perpendicular to $\vec{\beta}_2$. However, the theory developed in this text equally applies for lossy upper media.

Leaky waves violate the radiation condition since they only may exist if power is delivered to medium 1 from outside, in a direction towards the material interface with medium 2. This can be achieved by replacing the perfectly conducting plane in Figure 3.2 by the outer wall of a slotted waveguide. It also important to know that a leaky wave can not be exited by a plane wave incident from medium 2. In such a case, the result will be a standing wave in medium 2.

3.4 Plane Surface Waves along a Coated, Electric Perfectly Conducting Plane

3.4.1 Introduction

In the previous section the existence of surface waves was shown by making use of ray-optics theory, which is merely an approximate theoretical model. A rigorous approach consists of treating the layer structure as a boundary-value problem and solving it using Hertz potentials. A first analysis deals with the propagation of plane surface waves along the interface of a homogeneous linear isotropic half space with a homogeneous linear isotropic layer of finite height h that is supported by an electric perfectly conducting plane (Fig. 3.4). The solution of the more general three-layer structure with arbitrary material constants will be presented later in this chapter. The special case of an electric perfectly conducting substrate is presented first because it more readily provides the reader with a number of basic insights. It is important to note that the structures in the following sections can be solved both for forward propagating surface waves and leaky waves. For surface wave propagation it is necessary that $k_1 > k_2$. For leaky waves, k_1 may be smaller than k_2. Leaky waves violate the radiation condition and are therefore of little practical interest to RCS management.

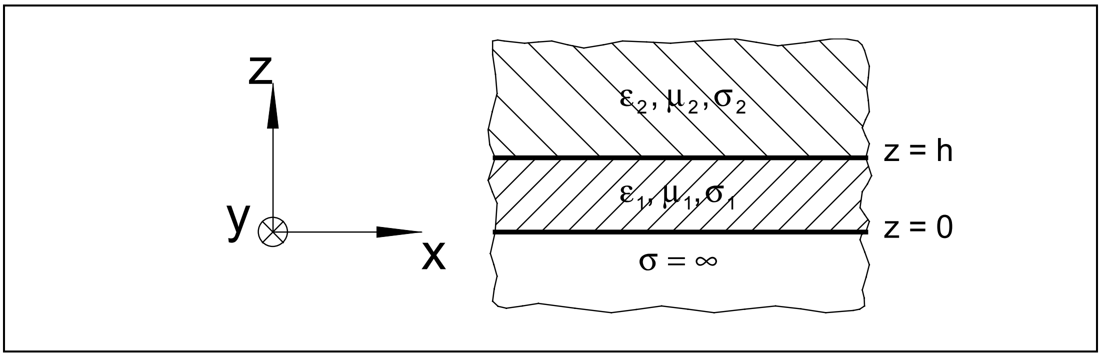


Figure 3.4: A coated, electric perfectly conducting plane; it is assumed that all media are homogeneous, linear and isotropic.

It is obvious that the Cartesian coordinate system (Section 2.4) is best suited for the analysis of plane waves. Assuming plane wave propagation in the x-direction, the structure of Figure 3.4 can be treated as a special case of a 2D-uniform guiding structure (see Section 2.5). Here however, none of the field components can have a y-dependence due to the fact that both media are infinite in the y-direction. Hence, the Hertz vector potential $\vec{\Pi}$ will have no y-dependence. Note that a field vector can still have components in the y-direction. For reasons that will be explained later, $\vec{\Pi}$ needs to be chosen in the z-direction.

A general expression for a Hertz vector potential having above-mentioned properties is

$$\vec{\Pi} = \Pi(z)e^{-j\beta_x x}\vec{e}_z \,. \tag{1}$$

In order to be able to apply (2.10) and (2.11), the relation between the curvilinear coordinates and the Cartesian coordinates must be as follows
$u_1 = z$; $u_2 = x$ and $u_3 = y$.

Substituting (1) into (2.10) results in general expressions for the field components of E-type waves within a medium

$$E_z = k^2 \Pi_e + \frac{\partial^2 \Pi_e}{\partial z^2}; \quad H_z = 0,$$

$$E_x = -j\beta_x \frac{\partial \Pi_e}{\partial z}; \quad H_x = (\sigma + j\omega\varepsilon)\frac{\partial \Pi_e}{\partial y} = 0, \tag{2}$$

$$E_y = \frac{\partial^2 \Pi_e}{\partial z \partial y} = 0; \quad H_y = j\beta_x(\sigma + j\omega\varepsilon)\Pi_e .$$

From (2) it can be seen that *E-type plane surface waves* are:

1) *longitudinal section magnetic* (LSM) waves; the magnetic field intensity $\vec{H}$ has no component in the direction normal to the material interface $(H_z = 0)$ and
2) *transversal magnetic* (TM) waves; the magnetic field intensity $\vec{H}$ has no component in the propagation direction $(H_x = 0)$.

Substituting (1) into (2.11) leads to general expressions for the field components of H-type waves within a medium

$$H_z = k^2 \Pi_m + \frac{\partial^2 \Pi_m}{\partial z^2}; \quad E_z = 0,$$

$$H_x = -j\beta_x \frac{\partial \Pi_m}{\partial z}; \quad E_x = -j\omega\mu \frac{\partial \Pi_m}{\partial y} = 0, \tag{3}$$

$$H_y = \frac{\partial^2 \Pi_m}{\partial z \partial y} = 0; \quad E_y = \beta_x \omega\mu \, \Pi_m \; .$$

It can be concluded from (3) that *H-type plane surface waves* are:

1) *longitudinal section electric* (LSE) waves; the electric field intensity $\vec{E}$ has no component in the direction normal to the material interface $(E_z = 0)$ and
2) *transversal electric* (TE) waves; the electric field intensity $\vec{E}$ has no component in the propagation direction $(E_x = 0)$.

3.4.2 E-Type Plane Surface Waves along a Coated, Electric Perfectly Conducting Plane

A suitable Hertz function for medium 1 that satisfies the boundary condition $E_x = 0$ at $z = 0$

is $\Pi_1 = A_1 \cos(s_{z1} z) e^{-j\beta_x x}$. (4)

The factor $\cos(s_{z1} z)$ may be interpreted as a standing wave in the z-direction.

Introducing (4) into (2) results in

$E_{z1} = A_1 \left(k_1^2 - s_{z1}^2\right) \cos(s_{z1} z) e^{-j\beta_x x}$, (5a)

$E_{x1} = j\beta_x A_1 s_{z1} \sin(s_{z1} z) e^{-j\beta_x x}$, (5b)

$E_{y1} = 0$, (5c)

$H_{z1} = 0$, (5d)

$H_{x1} = 0$, (5e)

$H_{y1} = j\beta_x (\sigma_1 + j\omega\varepsilon_1) A_1 \cos(s_{z1} z) e^{-j\beta_x x}$. (5f)

Recalling (2.13)

$s_{z1}^2 = k_1^2 - \beta_x^2 \;\Rightarrow\; s_{z1} = +\sqrt{k_1^2 - \beta_x^2}$. (6)

It is only for a matter of convenience that s_{z1} is chosen to equal the positive square root. Choosing the negative square root would have no effect on the results.

A suitable Hertz function for medium 2 that satisfies the boundary condition $\vec{E} = \vec{H} = \vec{0}$ when $z \to +\infty$

is $\Pi_2 = A_2 e^{-js_{z2}(z-h)} e^{-j\beta_x x}$. (7)

The factor $e^{-js_{z2}(z-h)}$ may be interpreted as a wave propagating in the positive z-direction with phase constant $s_{z2} = s'_{z2} - js''_{z2}$. Contrary to (6), the sign of s_{z2} is of importance here because s_{z2} belongs to the argument of an exponential function and therefore determines whether the solutions will be forward propagating surface waves or leaky waves. For surface waves, $s''_{z2} > 0 \;\Rightarrow\; \mathrm{Im}(s_{z2}) < 0$, which corresponds to a decaying field in the positive z-direction. If on the other hand $\mathrm{Im}(s_{z2}) > 0$, the wave is a leaky wave. In that case the radiation condition is violated because the field in medium 2 increases exponentially away from the interface.

The appropriate sign for s_{z2} can easily be found when both materials are lossless. k_2 and β_x are real numbers then. Moreover, all plane surface waves and leaky waves will be slow waves ($\beta_x > k_2$) as is the case for all inhomogeneous waves propagating in loss free media (see Appendix A).

For surface waves in loss free media, js_{z2} must be real and positive,
hence $js_{z2} = +\sqrt{-s_{z2}^2} = +\sqrt{\beta_x^2 - k_2^2}$
$\Rightarrow \; s_{z2} = -j\sqrt{\beta_x^2 - k_2^2} = +\sqrt{k_2^2 - \beta_x^2}$ (see also Appendix B),
whereas for leaky waves in loss free media
$s_{z2} = -\sqrt{k_2^2 - \beta_x^2} \;\Rightarrow\; js_{z2} = -j\sqrt{k_2^2 - \beta_x^2} = -\sqrt{\beta_x^2 - k_2^2}$.

However, things are more complicated when at least one of both media contains losses. Surface waves and leaky waves no longer need to be slow waves. The many possibilities for the value of s_{z2} will be discussed now for the case $k_2 = k_0$.

Thus, $s_{z0}^2 = k_0^2 - \beta_x^2 = k_0^2 - \left(\beta_x' - j\beta_x''\right)^2 = k_0^2 - \beta_x'^2 + \beta_x''^2 + 2j\beta_x'\beta_x''$

$$\Rightarrow \; \angle s_{z0}^2 = \operatorname{atan}\left(\frac{2\beta_x'\beta_x''}{k_0^2 - \beta_x'^2 + \beta_x''^2}\right).$$

Finally, de Moivre's theorem gives

$$\angle s_{z0} = \frac{1}{2}\operatorname{atan}\left(\frac{2\beta_x'\beta_x''}{k_0^2 - \beta_x'^2 + \beta_x''^2}\right) + p\pi$$

where p is either 0 or 1.

Now, let $u = \left(\frac{2\beta_x'\beta_x''}{k_0^2 - \beta_x'^2 + \beta_x''^2}\right)$.

The four possibilities for the location of s_{z0} in the s_{z0}-plane (Fig. 3.5) are

$u > 0, p = 0 \;\Rightarrow\; \angle s_{z0} \in \left]0, \frac{\pi}{4}\right[\;\Rightarrow$ a leaky wave,

$u \geq 0, p = 1 \;\Rightarrow\; \angle s_{z0} \in \left[\pi, \frac{5\pi}{4}\right[\;\Rightarrow$ a surface wave,

$u \leq 0, p = 0 \;\Rightarrow\; \angle s_{z0} \in \left]-\frac{\pi}{4}, 0\right] = \left]\frac{7\pi}{4}, 2\pi\right] \Rightarrow$ a surface wave,

$u < 0, p = 1 \;\Rightarrow\; \angle s_{z0} \in \left]\frac{3\pi}{4}, \pi\right[\;\Rightarrow$ a leaky wave.

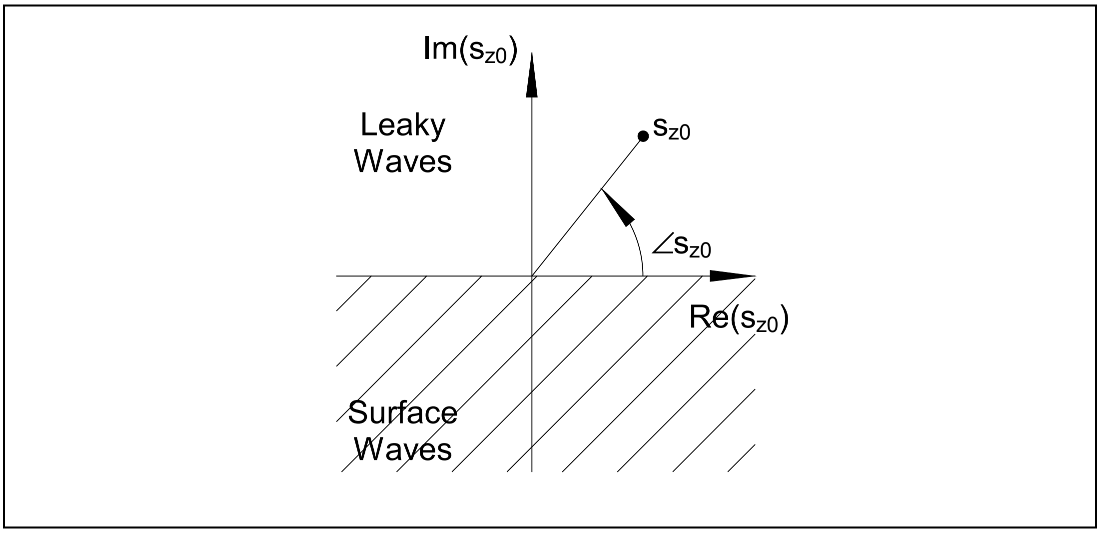


Figure 3.5: The complex s_{z0}-plane

Slow surface waves with moderate losses will most often fall into the third category. Equation (8a) gives rise to surface wave solutions as long as

$u \le 0 \;\Leftrightarrow\; k_0^2 - \beta_x'^2 + \beta_x''^2 \le 0 \;\Leftrightarrow\; \beta_x''^2 \le k_0^2 - \beta_x'^2$

where k_0, β_x' and β_x'' are all positive real numbers.

Even surface wave absorbers will almost always meet this requirement. This is shown by the numerical examples presented later in this chapter. However, to remain as general as possible, surface wave solutions are only obtained by letting $\mathrm{Re}(js_{z2}) \ge 0$ or

$$js_{z2} = \mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right]\sqrt{\beta_x^2 - k_2^2}\,. \tag{8a}$$

To obtain leaky wave solutions, let

$$js_{z2} = -\mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right]\sqrt{\beta_x^2 - k_2^2}\,. \tag{8b}$$

Introducing (7) into (2) leads to

$$E_{z2} = A_2\left(k_2^2 - s_{z2}^2\right)e^{-js_{z2}(z-h)}e^{-j\beta_x x}, \tag{9a}$$

$$E_{x2} = -\beta_x A_2 s_{z2} e^{-js_{z2}(z-h)}e^{-j\beta_x x}, \tag{9b}$$

$$E_{y2} = 0, \tag{9c}$$

$$H_{z2} = 0, \tag{9d}$$

$$H_{x2} = 0, \tag{9e}$$

$$H_{y2} = j\beta_x\left(\sigma_2 + j\omega\varepsilon_2\right)A_2 e^{-js_{z2}(z-h)}e^{-j\beta_x x}. \tag{9f}$$

The tangential components of both $\vec{E}$ and $\vec{H}$ are continuous across the interface of two media and therefore
$E_{x1} = E_{x2}$ at $z = h$

$$\Rightarrow \; A_1 s_{z1} \sin(s_{z1} h) = jA_2 s_{z2}, \tag{10}$$

as well as $H_{y1} = H_{y2}$ at $z = h$

$$\Rightarrow \; (\sigma_1 + j\omega\varepsilon_1) A_1 \cos(s_{z1} h) = (\sigma_2 + j\omega\varepsilon_2) A_2 . \tag{11}$$

Note that (10) and (11) would have resulted in a set of contradictory equations, if $\vec{\Pi}_{m,e}$ were chosen in any direction other than the z-direction.

Dividing (10) by (11) yields

$$\frac{s_{z1}}{\sigma_1 + j\omega\varepsilon_1} \tan(s_{z1} h) = \frac{js_{z2}}{\sigma_2 + j\omega\varepsilon_2} . \tag{12}$$

Substituting (6) and (8a) into (12) results in the following expression for E-type surface waves

$$\frac{\sqrt{k_1^2 - \beta_x^2}}{\sigma_1 + j\omega\varepsilon_1} \tan\left(h\sqrt{k_1^2 - \beta_x^2}\right) = \frac{\operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right]\sqrt{\beta_x^2 - k_2^2}}{\sigma_2 + j\omega\varepsilon_2} . \tag{13}$$

This equation is transcendental and can therefore only be solved numerically for β_x. It is called *a dispersion equation* because it expresses the nonlinear frequency dependence of β_x. Both equation (12) and (13) are expressions for *the transverse resonance condition* which requires the same value for the longitudinal wave impedance looking straight down to the interface (z = h) (14) as for the longitudinal wave impedance looking straight up [7, p. 12-6]. Hence, there will be no reflection in the equivalent transmission line of the layer structure (Fig. 3.7).

Looking straight down from medium 2 to the interface, *the longitudinal surface impedance* is (for a definition see Section 5.3.2)

$$Z_{s\ell} = -\frac{E_\ell}{H_t} = -\frac{E_{x2}(z = h)}{H_{y2}(z = h)} = -\frac{js_{z2}}{\sigma_2 + j\omega\varepsilon_2} = j\frac{js_{z2}}{\omega\varepsilon_2 - j\sigma_2} . \tag{14}$$

The minus sign in (14) originates from the fact that the Poynting vector $\vec{S} = \vec{E}_{x2} \times \vec{H}_{y2}$ is in the positive z-direction, whereas $Z_{s\ell}$ is the surface impedance at the interface, looking in the negative z-direction.

The value of the transversal surface impedance is undefined.

3.4.3 H-Type Plane Surface Waves along a Coated, Electric Perfectly Conducting Plane

A suitable Hertz function for medium 1 that satisfies the boundary condition $E_y = 0$ at $z = 0$

is $\Pi_1 = A_1 \sin(s_{z1}z)e^{-j\beta_x x}$. (15)

The factor $\sin(s_{z1}z)$ may be interpreted as a standing wave in the z-direction.

Introducing (15) into (3) results in

$H_{z1} = A_1\left(k_1^2 - s_{z1}^2\right)\sin(s_{z1}z)e^{-j\beta_x x}$, (16a)

$H_{x1} = -j\beta_x A_1 s_{z1}\cos(s_{z1}z)e^{-j\beta_x x}$, (16b)

$H_{y1} = 0$, (16c)

$E_{z1} = 0$, (16d)

$E_{x1} = 0$, (16e)

$E_{y1} = \beta_x \omega\mu_1 A_1 \sin(s_{z1}z)e^{-j\beta_x x}$. (16f)

Recalling (2.13)

$s_{z1}^2 = k_1^2 - \beta_x^2 \;\Rightarrow\; s_{z1} = +\sqrt{k_1^2 - \beta_x^2}$. (17)

It is only for a matter of convenience that s_{z1} is chosen to equal the positive square root. Choosing the negative square root would have no effect on the results.

A suitable Hertz function for medium 2 that satisfies the boundary condition $\vec{E} = \vec{H} = \vec{0}$ when $z \to +\infty$

is $\Pi_2 = A_2 e^{-js_{z2}(z-h)}e^{-j\beta_x x}$. (18)

For s_{z2}, the same reasoning applies as in the previous section.
Hence, surface wave solutions are obtained by letting $\mathrm{Re}(js_{z2}) \geq 0$ or

$js_{z2} = \mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right]\sqrt{\beta_x^2 - k_2^2}$. (19a)

To obtain leaky wave solutions, let

$js_{z2} = -\mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right]\sqrt{\beta_x^2 - k_2^2}$. (19b)

Introducing (18) into (3) leads to

$H_{z2} = A_2\left(k_2^2 - s_{z2}^2\right)e^{-js_{z2}(z-h)}e^{-j\beta_x x}$, (20a)

$H_{x2} = -\beta_x A_2 s_{z2} e^{-js_{z2}(z-h)}e^{-j\beta_x x}$, (20b)

$H_{y2} = 0$, (20c)

$E_{z2} = 0$, (20d)

$E_{x2} = 0$, (20e)

$E_{y2} = \beta_x \omega\mu_2 A_2 e^{-js_{z2}(z-h)}e^{-j\beta_x x}$. (20f)

The tangential components of both $\vec{E}$ and $\vec{H}$ are continuous across the interface of two media and therefore
$H_{x1} = H_{x2}$ at $z = h$

$$\Rightarrow \; A_1 s_{z1} \cos(s_{z1} h) = -jA_2 s_{z2}, \tag{21}$$

as well as $E_{y1} = E_{y2}$ at $z = h$

$$\Rightarrow \; \mu_1 A_1 \sin(s_{z1} h) = \mu_2 A_2 . \tag{22}$$

Note that (22) and (21) would have resulted in a set of contradictory equations, if $\vec{\Pi}$ were chosen in any direction other than the z-direction.

Dividing (22) by (21) and multiplying both sides by $j\omega$ yields

$$\frac{j\omega\mu_1}{s_{z1}} \tan(s_{z1} h) = -\frac{j\omega\mu_2}{js_{z2}} . \tag{23}$$

Substituting (17) and (19a) into (23) results in the following expression for H-type surface waves

$$\frac{j\omega\mu_1}{\sqrt{k_1^2 - \beta_x^2}} \tan\left(h\sqrt{k_1^2 - \beta_x^2}\right) = -\frac{j\omega\mu_2}{\operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right]\sqrt{\beta_x^2 - k_2^2}} . \tag{24}$$

This dispersion equation is transcendental and can therefore only be solved numerically for β_x. Both equation (23) and (24) are expressions for the transverse resonance condition which requires the same value for the transversal wave impedance looking straight down to the interface (z = h) (25) as for the transversal wave impedance looking straight up [7, p. 12-6]. Hence, there will be no reflection in the equivalent transmission line of the layer (Fig. 3.7).

Looking straight down from medium 2 to the interface, *the transversal surface impedance* is (for a definition see Section 5.3.2)

$$Z_{st} = \frac{E_t}{H_\ell} = \frac{E_{y2}(z = h)}{H_{x2}(z = h)} = -\frac{\omega\mu_2}{s_{z2}} = -j\frac{\omega\mu_2}{js_{z2}} . \tag{25}$$

In (25), the Poynting vector $\vec{S} = \vec{E}_{y2} \times \vec{H}_{x2}$ is in the negative z-direction, the same direction used for determining Z_{st}. Hence, no change in sign is needed as in (14).

The value of the longitudinal surface impedance is undefined.

3.4.4 High-Frequency Solution for E-Type and H-Type Plane Surface Waves along a Coated, Electric Perfectly Conducting Plane

The transcendental equations (12) and (23) will now be solved in the limit case when the frequency $f \to +\infty$. The characteristics of a surface wave are primarily determined by the quantities β_x and js_{z2}, the phase constant in the propagation direction and the decay in the direction perpendicular to the material interface, respectively.

Applying (2.13) twice gives the relation between s_{z2} and s_{z1}

$$s_{z1}^2 - s_{z2}^2 = k_1^2 - \beta_x^2 - k_2^2 + \beta_x^2 = k_1^2 - k_2^2$$

$$\Rightarrow \; s_{z1}^2 = k_1^2 - k_2^2 + s_{z2}^2 . \qquad (27)$$

$s_{z1}h$ appears as the argument of a tangent function in the dispersion equation of both E-type as H-type surface waves. A tangent function takes on every positive value in the interval [0,π/2[and every negative value in]π/2,π]. Consequently, the first root of dispersion equation (12), which corresponds to the fundamental E-type mode, occurs at $0 < s_{z1}h < \pi/2$ $\Rightarrow \; 0 < s_{z1} < \pi/2h$. Similarly, the first root of dispersion equation (23), which corresponds to the fundamental H-type mode, occurs at $\pi/2 < s_{z1}h < \pi \;\; \Rightarrow \; \pi/(2h) < s_{z1} < \pi/h$. So for both wave types, s_{z1} will always have a finite value.

By contrast, $k_1^2 - k_2^2$ will become infinite if $f \to +\infty$ because $k_1^2 - k_2^2 = (\varepsilon_{r1}\mu_{r1} - \varepsilon_{r2}\mu_{r2})\varepsilon_0\mu_0\omega^2$.

Knowing the behaviour of s_{z1} and $k_1^2 - k_2^2$ for $f \to +\infty$, equation (27) must result in $s_{z2}^2 \to -\infty \; \Rightarrow \; js_{z2} \to +\infty$ for $f \to +\infty$. This means that the surface wave field will not extend outside the coating layer for extremely high frequencies. Optical dielectric waveguides work on this principle.

Also, $s_{z1}^2 << k_1^2$ for $f \to +\infty$, because, as was shown before, s_{z1} remains bounded for very high frequencies.

Hence, $\beta_x^2\big|_{f\to+\infty} \equiv \left(k_1^2 - s_{z1}^2\right)\Big|_{f\to+\infty} = k_1^2\big|_{f\to+\infty}$. (28)

Thus at extremely high frequencies a surface wave behaves as a inhomogeneous plane wave propagating entirely in medium 1. In general, the wave will remain inhomogeneous because s_{z1} does not have to be zero in (5b) and (16b).

3.4.5 Low-Frequency Solution for E-Type Plane Surface Waves along Thin Coatings on a Plane PEC

When the frequency is very low, the wave number k_1 will be very small. Since $s_{z1} = \sqrt{k_1^2 - \beta_x^2}$, s_{z1} will also be very small. The tangent function in equation (12) can be approximated by its argument $s_{z1}h$ if also h is electrically small, but not necessarily zero.

If this is the case, the dispersion equation for E-type surface waves (12) reduces to

$$\left.\frac{s_{z1}^2 h}{\sigma_1 + j\omega\varepsilon_1}\right|_{f\to 0} = \left.\frac{js_{z2}}{\sigma_2 + j\omega\varepsilon_2}\right|_{f\to 0}. \tag{29}$$

Raising the power of (29), substituting (27) and rewriting gives

$$h^2\left(\sigma_2 + j\omega\varepsilon_2\right)^2 \left.s_{z1}^2\right|_{f\to 0} = \left.\left(\sigma_2 + j\omega\varepsilon_2\right)^2\left(k_1^2 - k_2^2 - s_{z1}^2\right)\right|_{f\to 0}$$

$$\Rightarrow\ \left.s_{z1}^2\right|_{f\to 0} = \left.\frac{\left(\sigma_1 + j\omega\varepsilon_1\right)^2\left(k_1^2 - k_2^2\right)}{\left(\sigma_1 + j\omega\varepsilon_1\right)^2 + h^2\left(\sigma_2 + j\omega\varepsilon_2\right)^2}\right|_{f\to 0}. \tag{30}$$

Substitute (30) into (6) to get

$$\left.\beta_x\right|_{f\to 0} = \left.\sqrt{k_1^2 - \frac{\left(\sigma_1 + j\omega\varepsilon_1\right)^2\left(k_1^2 - k_2^2\right)}{\left(\sigma_1 + j\omega\varepsilon_1\right)^2 + h^2\left(\sigma_2 + j\omega\varepsilon_2\right)^2}}\right|_{f\to 0}$$

$$\Rightarrow\ \left.\beta_x\right|_{f\to 0} = \left.\sqrt{\frac{h^2\left(\sigma_2 + j\omega\varepsilon_2\right)^2 k_1^2 + \left(\sigma_1 + j\omega\varepsilon_1\right)^2 k_2^2}{\left(\sigma_1 + j\omega\varepsilon_1\right)^2 + h^2\left(\sigma_2 + j\omega\varepsilon_2\right)^2}}\right|_{f\to 0} \tag{31}$$

Moreover, if the coating is extremely thin $(h \to 0)$, (31) simplifies to

$$\left.\beta_x\right|_{\substack{f\to 0\\ h\to 0}} = \left.k_2\right|_{f\to 0} \tag{32}$$

and $js_{z2} \to 0$.

This means that at very low frequencies and when the coating is extremely thin, the propagating wave will no longer be an inhomogeneous plane surface wave but a homogeneous plane TEM-wave propagating entirely in medium 2. The wave will be homogeneous in this limit case because along a perfect electric conductor (PEC), the tangential components of an E-field are always zero.

A similar solution for H-type surface-waves along an electrically thin coating on a perfectly plane conductor does not exist. An explanation for this will be given in the next section.

3.4.6 Some Properties of Plane Surface Waves along a Coated, Electric Perfectly Conducting Plane

An important feature of surface waves is the fact that the type of surface wave that will propagate along a coated structure, is entirely determined by the surface impedance at the material interface of the structure.

For E-type surface waves, the expression found for the longitudinal surface impedance was (14)

$$Z_{s\ell} = j\frac{js_{z2}}{\omega\varepsilon_2 - j\sigma_2}, \text{ which is} \geq 0 .$$

This implies that the longitudinal surface impedance has to be inductive for E-type surface waves to propagate along a coated PEC.
Expression (14) can be rewritten, by making use of equation (12), in a form comparable with the input impedance of a shorted transmission line [6, p. 503]

$$Z_{s\ell} = -\frac{s_{z1}}{\sigma_1 + j\omega\varepsilon_1}\tan(s_{z1}h) = j\frac{s_{z1}}{\omega\varepsilon_1 - j\sigma_1}\tan(s_{z1}h) . \qquad (33)$$

Therefore, in order to obtain an inductive surface impedance, the electrical height of the coating must be such that

$$n\pi \leq s_{z1}h < (2n+1)\frac{\pi}{2} \qquad (34)$$

where n is a positive integer and $s_{z1} = \dfrac{2\pi}{\lambda_{z1}}$.
n+1 is also the total number of modes that may exist along a PEC with a given coating of height h. Note that only the fundamental E-type mode has no low-frequency cutoff. It is worth pointing out that below the cutoff frequency a surface wave does not become evanescent but ceases to exist altogether [7, p. 12-7].

The only wave able to propagate along the coated structure is a vertically polarized TEM-wave when $s_{z1}h = n\pi$. This wave can be seen as a degenerate form of an E-type surface wave. The phase constant β_x will equal k_2 in this case and $js_{z2} \to 0$.

Likewise, the expression for the transversal surface impedance associated with H-type surface waves is (25)

$$Z_{st} = -j\frac{\omega\mu_2}{js_{z2}}, \text{ which is} < 0.$$

This means that the transversal surface impedance has to be capacitive in order to have H-type surface wave propagation.
By virtue of (23), (25) becomes

$$Z_{st} = j\frac{\omega\mu_1}{s_{z1}}\tan(s_{z1}h). \tag{35}$$

Hence, the constraints for the height h of the coating are

$$(2n+1)\frac{\pi}{2} \le s_{z1}h < (n+1)\pi \tag{36}$$

where n is a positive integer and $s_{z1} = \dfrac{2\pi}{\lambda_{z1}}$.

n+1 is also the total number of modes that may exist along a PEC with a given coating of height h. Note that all H-type modes, even the lowest order mode, have low-frequency cutoff. This is why one should refrain from calling the lowest H-type mode the fundamental H-type mode. However, the lowest order E-type surface wave mode *is* the fundamental mode. All this can be explained by the fact that this study deals with surface waves along a coated, electric perfectly conducting plane and not a coated, magnetic perfectly conducting plane.

The only wave able to propagate along the coated structure is a horizontally polarized TEM-wave when $s_{z1}h = (2n+1)\dfrac{\pi}{2}$.

This wave can be seen as a degenerate form of an H-type surface wave. The phase constant β_x will equal k_2 in this case and $js_{z2} \to 0$.

In the case of a coated plane PEC, the fundamental E-type mode is usually called the TM_0 mode. Whereas the lowest H-type is labelled as the TE_1 mode. This somewhat peculiar numbering system originates from the mode numbering in plane dielectric slab waveguides [3, pp. 712-716].

In contrast to ordinary metallic waveguides, only a finite number of discrete surface wave modes (i.e. n+1 modes) may exist at any given frequency.
As shown in the preceding sections it is necessary that $k_1 > k_2$ for surface wave propagation to occur.

Both E-type and H-type surface waves can also be supported by a corrugated surface with thin metal walls and a suitable artificial surface impedance [3, pp. 708-712]. Corrugated surfaces with thick metal walls will briefly be discussed in Chapter 5.

3.4.7 The Continuous Eigenvalue Spectrum and Improper Solutions

Guiding structures may be classified into closed and open types. *A closed guiding structure* possesses a finite cross section which is bounded by wall that are impermeable to radiation and confine the electromagnetic field to the interior of the waveguide. The field inside a closed guiding structure may be decomposed into *a complete set of discrete normal modes* each of which individually satisfies the relevant boundary conditions [8, p.155].

In contrast to closed waveguides, *open guiding structures* do not possess walls completely impermeable to radiation and, therefore, power flow and stored energy are not confined to the inside of the guiding structure*. The radiated field is represented by a continuous spectrum of modes* which on open structures appears in addition to the discrete mode spectrum. *The continuous eigenvalue spectrum* of planar structures consist of all homogeneous and inhomogeneous standing plane waves that individually satisfy the boundary conditions with a continuous range of phase constants such that $-\infty < \beta_x^2 \leq k_2^2$ [8, p. 156]. On the other hand, the discrete spectrum (also termed proper) contains only a finite number of modes that decay at infinity. *The proper discrete eigenvalue spectrum* corresponds to the various surface waves supported by the structure and which are solutions of the dispersion equations (12) and (23). In contrast to a closed guiding structure, the dispersion equations of an open structure may possess, in addition to the proper solutions, other discrete solutions, termed improper, that correspond to fields which grow away from the structure and violate the radiation condition. *The improper discrete eigenvalue spectrum* represents the various leaky waves which are, as was mentioned before, improper solutions to the dispersion equations. The totality of the proper discrete eigenvalue spectrum and the continuous eigenvalue spectrum corresponds to a complete set of eigenfunctions along which the physical field along an open structure may be expanded.

Note that in the previous discussion the more general term “eigenvalue” is used instead of the word “root”. Here is explained why. Roots are solutions to a dispersion equation, whereas eigenvalues are solutions to Hertz’s vector wave equation (2.1), which is in fact an eigenvalue equation. All roots of a dispersion equation are eigenvalues of (2.1), but not all eigenvalues of (2.1) are roots of a dispersion equation.

To find the complete set of eigenvalues in which the field of an arbitrary source may be expanded, consider a plane wave, not necessary homogeneous, incident at an angle θ_{2i} on a coated PEC, as shown in Figure 3.6 for a parallel polarized plane wave. Remember that an inhomogeneous plane wave is represented by an imaginary angle of incidence.

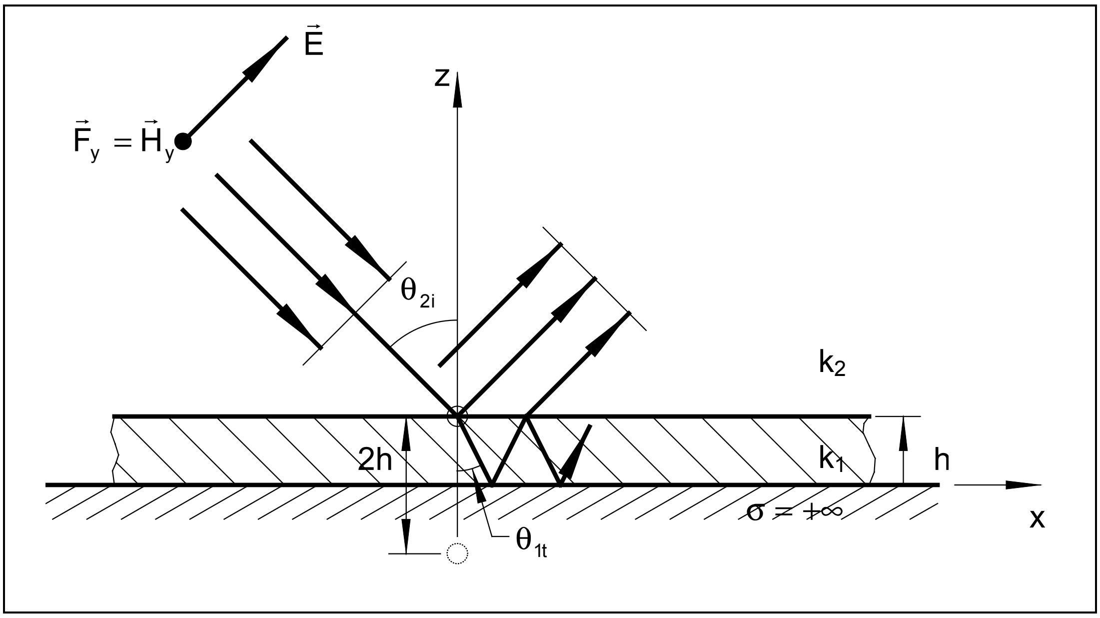


Figure 3.6: A parallel polarized plane wave incident at an angle θ_i on a coated PEC

In the coating, a standing wave may exist due to the reflections at the electric perfectly conducting plane and the material interface. There may also be a reflected wave above the coating. If this is the case, the resulting field above the interface will be that of a standing wave. Note that in these statements no restrictions are put on the value of the wave numbers k_1 and k_2. This means that even if $k_2 > k_1$, the resulting field in medium 2 will still be a standing wave, not a leaky wave. At this point, it is interesting to compare the situation in Figure 3.6 with that of Figure 3.2 where in medium 2 a leaky wave will exist when $k_2 > k_1$. The big difference between Figure 3.6 and Figure 3.2 is that in Figure 3.6 the incident wave comes from medium 2 while in Figure 3.2 the wave is incident from medium 1. This explains the absence of leaky wave modes in Figure 3.6, even if $k_2 > k_1$.

The transverse field F_y in the two regions may be represented as follows

$$F_{y2} = A_2 \exp\left[-jk_2\left[-z\cdot\cos(\theta_{2i}) + x\cdot\sin(\theta_{2i})\right]\right]$$

$$+RA_2 \exp\left[-jk_2\left[(z-2h)\cos(\theta_{2i}) + x\cdot\sin(\theta_{2i})\right]\right], \quad (37a)$$

$$F_{y1} = A_1 \cos\left[k_1 z\cdot\cos(\theta_{1t})\cdot\exp\left(-jk_1 x\cdot\sin(\theta_{1t})\right)\right] \quad (37b)$$

where

$F_y = E_y$ for perpendicular polarized waves and

$F_y = H_y$ for parallel polarized waves.

It may be necessary to shed some light on the origin of these expressions. The field in medium 2 is written explicitly as the combination of an incident

wave and a reflected wave. The complex amplitude ratio between the reflected and the incident wave is given by *the reflection coefficient* R. The field in medium 1 is a standing wave. In both media the wave vector $\vec{k}$ is decomposed into its components along the x- and z-axis
$\vec{k}_x = \vec{k}\sin(\theta)$; $\vec{k}_z = \vec{k}\cos(\theta)$.

Note that in (37) the phase is referenced to the phase at the point (0,0,h). Hence, the phase of the reflected wave equals zero in the image point (0,0,-h).

The boundary conditions require the tangential field intensities to be continuous across the material interface. They can easily be imposed by making use of *a transverse equivalent network* (Fig. 3.7) [8, pp. 156-162]. This eliminates the need to deal with the field expressions (37) directly. The dispersion equations and hence the discrete eigenvalue spectrum can be obtained by applying the transverse resonance. *The transverse resonance condition* requires that at any point along the equivalent transmission line, the sum of the impedance looking in one direction and the impedance looking into the other direction equals zero or
$Z\uparrow + Z\downarrow = 0$ which is equivalent to $Y\uparrow + Y\downarrow = 0$ [8, p. 158]. (38)

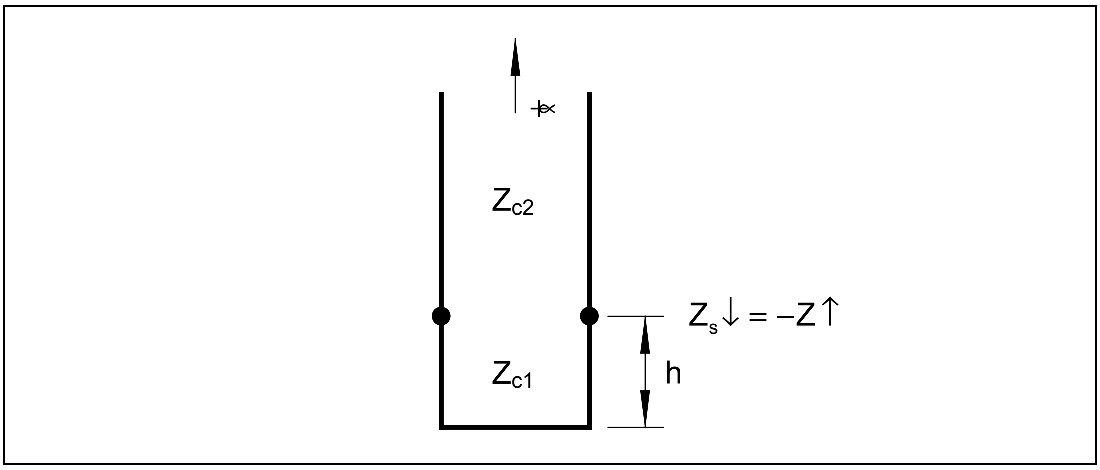


Figure 3.7: Transmission-line equivalent of a coated PEC

In Figure 3.7, Z_{c1} is the characteristic wave impedance of the coating and Z_{c2} the characteristic wave impedance of medium 2. For parallel polarized incident waves these characteristic impedances are

$$Z_{c1//} = \frac{E_{x1}}{H_{y1}} = \eta_1\cos(\theta_{1t}) \text{ and } Z_{c2//} = \frac{E_{x2}}{H_{y2}} = \eta_2\cos(\theta_{2i}), \quad (39)$$

whereas for perpendicular polarized incident waves

$$Z_{c1\perp} = \frac{E_{y1}}{H_{x1}} = \frac{\eta_1}{\cos(\theta_{1t})} \text{ and } Z_{c2\perp} = \frac{E_{y2}}{H_{x2}} = \frac{\eta_2}{\cos(\theta_{2i})}. \quad (40)$$

The coating acts like a length of transmission line terminated by a short circuit (Fig. 3.7). Hence, for parallel polarized waves the surface impedance at height z = h is

$$Z_{s\ell//} = \frac{E_x(z=h)}{H_y(z=h)} = jZ_{c1//}\tan\left[hk_1\cos(\theta_{1t})\right] = j\eta_1\cos(\theta_{1t})\tan\left[hk_1\cos(\theta_{1t})\right], \tag{41}$$

whereas for perpendicular polarized waves

$$Z_{st\perp} = \frac{E_y(z=h)}{H_x(z=h)} = jZ_{c1\perp}\tan\left[hk_1\cos(\theta_{1t})\right] = j\frac{\eta_1}{\cos(\theta_{1t})}\tan\left[hk_1\cos(\theta_{1t})\right]. \tag{42}$$

Note that the surface impedance is a longitudinal impedance for parallel polarized waves and a transversal impedance for perpendicular polarized waves.

Applying the transverse resonance condition to (39) and (41), respectively (40) and (42), results in the dispersion equations for E-type and H-type surface waves, respectively. Moreover, the proper solutions to these dispersion equations are poles of the reflection coefficient R, as will be shown now.

For parallel polarized waves, $R_{//}$ equals the current reflection coefficient Γ_i because $F_y = H_y$ in (37a), thus

$$R_{//} = \Gamma_{i//} = \frac{Z_{c2//} - Z_{s\ell//}}{Z_{c2//} + Z_{s\ell//}} = \frac{\eta_2\cos(\theta_{2i}) - j\eta_1\cos(\theta_{1t})\tan\left[hk_1\cos(\theta_{1t})\right]}{\eta_2\cos(\theta_{2i}) + j\eta_1\cos(\theta_{1t})\tan\left[hk_1\cos(\theta_{1t})\right]}. \tag{43}$$

For perpendicular polarized waves, $F_y = E_y$ and hence $R_\perp$ must equal the voltage reflection coefficient Γ_v

$$R_\perp = \Gamma_{v\perp} = \frac{Z_{st\perp} - Z_{c2\perp}}{Z_{st\perp} + Z_{c2\perp}} = \frac{j\dfrac{\eta_1}{\cos(\theta_{1t})}\tan\left[hk_1\cos(\theta_{1t})\right] - \dfrac{\eta_2}{\cos(\theta_{2i})}}{j\dfrac{\eta_1}{\cos(\theta_{1t})}\tan\left[hk_1\cos(\theta_{1t})\right] + \dfrac{\eta_2}{\cos(\theta_{2i})}}. \tag{44}$$

A first group of solutions to (37a) and (37b) are the E-type surface wave modes which are poles for the reflection coefficient $R_{//}$. To see this, apply the transverse resonance condition to (43)

$$R_{//} \to \infty \iff Z_{c2//} + Z_{s\ell//} = 0 \iff \eta_2 \cos(\theta_{2i}) + j\eta_1 \cos(\theta_{1t}) \tan\left[hk_1 \cos(\theta_{1t})\right] = 0. \tag{45}$$

In (45), let

$$s_{z1} = k_1 \cos(\theta_{1t}) \tag{46}$$

$$\Rightarrow \; j\eta_1 \cos(\theta_{1t}) = j\frac{\eta_1}{k_1} s_{z1} = \frac{j\sqrt{\dfrac{\mu_1}{\varepsilon_1 - j\dfrac{\sigma_1}{\omega}}}}{\omega\sqrt{\mu_1\left(\varepsilon_1 - j\dfrac{\sigma_1}{\omega}\right)}} s_{z1} = -\frac{s_{z1}}{\sigma_1 + j\omega\varepsilon_1} \tag{47}$$

and $s_{z2} = k_2 \cos(\theta_{2i}) \;\Rightarrow\; \eta_2 \cos(\theta_{2i}) = \dfrac{js_{z2}}{\sigma_2 + j\omega\varepsilon_2}$. (48)

Substituting (46), (47) and (48) into (45) results in

$$\frac{s_{z1}}{\sigma_1 + j\omega\varepsilon_1} \tan(s_{z1}h) = \frac{js_{z2}}{\sigma_2 + j\omega\varepsilon_2}.$$

This is the dispersion equation for E-type surface wave modes (12) which was derived earlier in Section 3.4.2. The here presented alternative method for finding a dispersion equation might be somewhat quicker, it does not provide the insight into the actual field distributions of the propagating wave.

H-type surface wave modes are also solutions to (37a) and (37b) and at the same time poles for the reflection coefficient $R_\perp$. To see this, apply the transverse resonance condition to (44)

$$R_\perp \to \infty \;\Leftrightarrow\; Z_{st\perp} + Z_{c2\perp} = 0 \;\Leftrightarrow\; j\frac{\eta_1}{\cos(\theta_{1t})}\tan\left[hk_1\cos(\theta_{1t})\right] + \frac{\eta_2}{\cos(\theta_{2i})} = 0. \tag{49}$$

In (49), let

$$s_{z1} = k_1\cos(\theta_{1t}) \tag{50}$$

$$\Rightarrow\; j\frac{\eta_1}{\cos(\theta_{1t})} = j\frac{k_1\eta_1}{s_{z1}} = j\omega\sqrt{\mu_1\left(\varepsilon_1 - j\frac{\sigma_1}{\omega}\right)}\sqrt{\frac{\mu_1}{\varepsilon_1 - j\frac{\sigma_1}{\omega}}}\frac{1}{s_{z1}} = \frac{j\omega\mu_1}{s_{z1}} \tag{51}$$

and $s_{z2} = k_2\cos(\theta_{2i}) \;\Rightarrow\; \dfrac{\eta_2}{\cos(\theta_{2i})} = \dfrac{j\omega\mu_2}{js_{z2}}$. (52)

Substituting (50), (51) and (52) into (49) results in

$$\frac{j\omega\mu_1}{s_{z1}}\tan(s_{z1}h) = -\frac{j\omega\mu_2}{js_{z2}}.$$

This is the dispersion equation for H-type surface wave modes (23) which was derived earlier in Section 3.4.3.

Although up to now surface waves have been considered to be produced by the pole in the expression for the reflection coefficient, they may equally be seen as being produced by a zero of the reflection coefficient. Whether a surface wave mode should be associated with a zero or a pole of the reflection coefficient depends on the fact whether the field above the dielectric is considered to be a reflected wave or an incident wave, because in case of the latter, (48) and (52) change to

$$s_{z2} = -k_2\cos(\theta_{2i}) \;\Rightarrow\; \eta_2\cos(\theta_{2i}) = -\frac{js_{z2}}{\sigma_2 + j\omega\varepsilon_2}$$

and $s_{z2} = -k_2\cos(\theta_{2i}) \;\Rightarrow\; \dfrac{\eta_2}{\cos(\theta_{2i})} = -\dfrac{j\omega\mu_2}{js_{z2}}$, respectively.

The characteristic wave impedances $Z_{c2//}$ and $Z_{c2\perp}$ change accordingly, but the dispersion equation remains the same.
To summarize, if the field in medium 2 is considered as an incident wave, the solution corresponds to a zero of the reflection coefficient and not a pole, as is the case if the field is considered as a reflected wave [3, p.718].

The system of equations (37a) and (37b) has also other solution types in addition to the previously found surface wave solutions. Standing waves along the z-axis exists for all real angles of incidence $0 \le \theta_{2i} \le 90^\circ$

$$\Rightarrow \ 0 \le s_{z2}^2 = k_2^2 \cos^2(\theta_{2i}) \le k_2^2$$
$$\Rightarrow \ 0 \le \beta_x^2 = k_2^2 - s_{z2}^2 \le k_2^2$$
$$\Rightarrow \ 0 \le \beta_x \le k_2 . \qquad (53)$$

A standing wave solution along the z-axis will also exist for imaginary angles of incidence

$$\left.\begin{aligned} s_{z2}^2 = k_2^2 \cos^2(\theta_{2i}) > k_2^2 \\ \beta_x^2 = k_2^2 - s_{z2}^2 \end{aligned}\right\} \Rightarrow \ -j\infty < \beta_x < j0 . \qquad (54)$$

The homogeneous and inhomogeneous standing wave solutions given in (53) and (54), respectively, form the continuous eigenvalue spectrum.

3.4.8 Traveling Waves Categorized by Surface Impedance

Both the proper and the improper discrete eigenvalue spectra represent guided wave modes, which are often called traveling waves. These plane waves are characterized by the orientation of $\vec{\alpha}_2$ and $\vec{\beta}_2$ in medium 2, which in turn depends on the sign of the real and imaginary parts of the surface impedance [8, pp. 164-167][1]. This relation will be explored in more detail now.

The complex wave vector $\vec{k}_2$ in medium 2 can be decomposed into its components along the x- and the z-axis

$\vec{k}_{x2} = \vec{k}_2 \sin(\theta_{2i})$; $\vec{k}_{z2} = \vec{k}_2 \cos(\theta_{2i})$

where $k_{z2} = \beta_{z2} - j\alpha_{z2} = s_{z2}$. (55)

Substituting (55) into the expression for the surface impedance previously obtained for E-type plane surface waves (14), gives

$$Z_{s\ell} = R_{s\ell} + jX_{s\ell} = -\frac{js_{z2}}{\sigma_2 + j\omega\varepsilon_2} = -\frac{s_{z2}}{\omega\varepsilon_2 - j\sigma_2} = \frac{-\beta_{z2} + j\alpha_{z2}}{\omega\varepsilon_2 - j\sigma_2}. \quad (56)$$

This results in the following set of rules for parallel (TM) polarized (E-type) traveling waves

$\mathrm{sign}(\beta_{z2}) = -\mathrm{sign}(R_{s\ell})$, (57a)

$\mathrm{sign}(\alpha_{z2}) = \mathrm{sign}(X_{s\ell})$. (57b)

Rewriting the surface impedance expression for H-type plane surface waves (25) as a surface admittance and substituting (55) yields

$$Y_{st} = G_{st} + jB_{st} = -\frac{s_{z2}}{\omega\mu_2} = \frac{-\beta_{z2} + j\alpha_{z2}}{\omega\mu_2}. \quad (58)$$

Hence, for perpendicular (TE) polarized (H-type) traveling waves

$\mathrm{sign}(\beta_{z2}) = -\mathrm{sign}(G_{st})$ and (59a)

$\mathrm{sign}(\alpha_{z2}) = \mathrm{sign}(B_{st})$. (59b)

[1] In reference [8], a rather unusual sign convention is used in which -j is replaced by +i.

The classification of traveling wave types according to the surface impedance is shown in Figure 3.8 and Figure 3.9.
(For more information, see [8, pp. 166-167].)

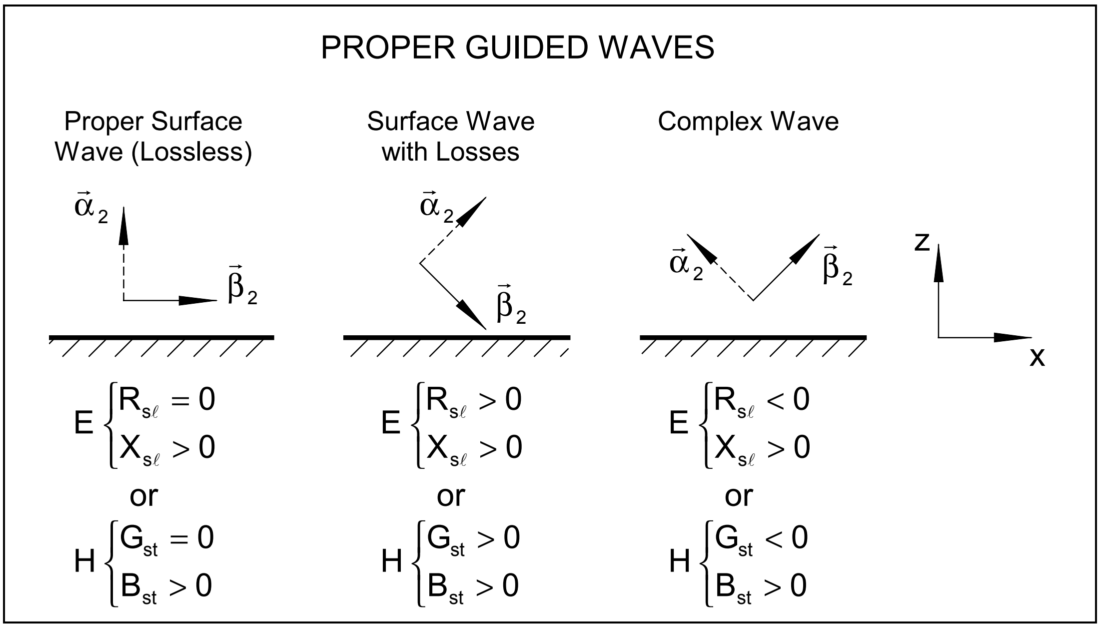


Figure 3.8: Classification of the proper guided waves along a coated plane PEC ($\mathrm{Im}(k_2) = 0$)

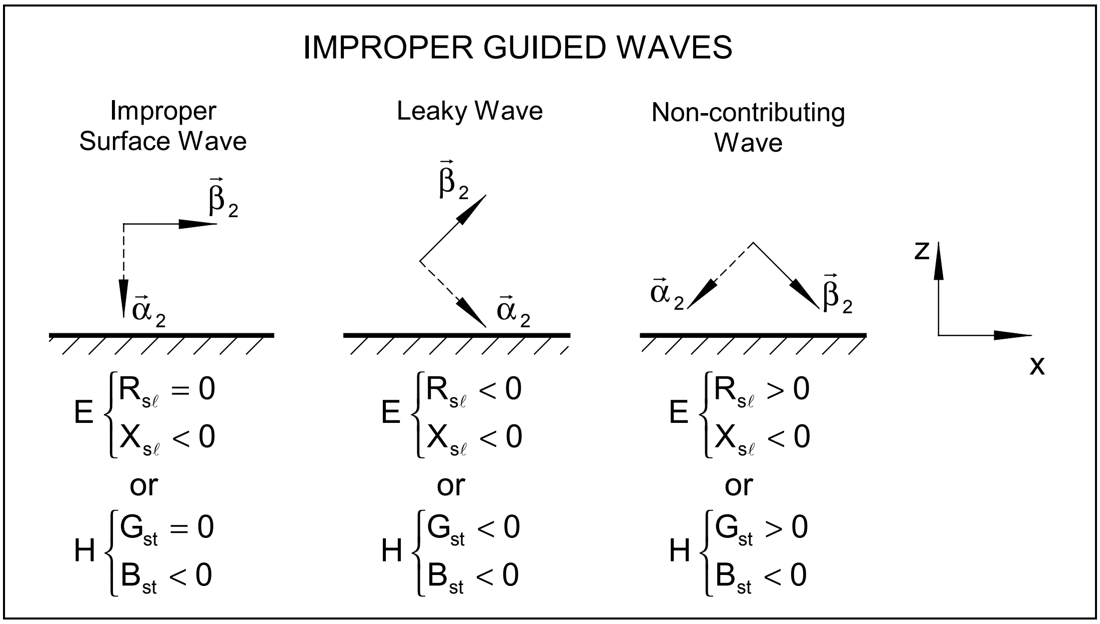


Figure 3.9: Classification of the improper guided waves along a coated plane PEC ($\mathrm{Im}(k_2) = 0$)

3.4.9 The Mapping of Fast and Slow Traveling Waves onto the w-Plane

Up to now, little has been said about the requirements for fast and slow traveling wave propagation. This is mainly due to the fact that s_{z2} has thus far always been expressed as a dual-valued function of β_x. This section explains how s_{z2} can be transformed into a single-valued function of a new complex variable w.

Fast electromagnetic waves are waves with a phase velocity v_p greater than c_0, whereas *slow electromagnetic waves* are waves with v_p smaller than c_0. c_0 is the velocity of light in a vacuum, i.e. 299792458m/s.

The phase velocity of a wave is

$$v_p = \frac{\mathrm{Re}(\beta)}{\omega}. \tag{60}$$

In view of (60), alternative definitions for fast and slow waves are
$\beta < k_0$ and $\beta > k_0$, respectively.
Note that it is only useful to talk about fast and slow waves when $k_2 = k_0$.

Introducing the trigonometric transformation (Fig. 3.10) [8, pp. 171 & 241]

$$\beta_x = k_0 \sin(w) \quad \text{where } w = \xi + j\eta, \tag{61}$$

results in a single-valued expression for s_{z0}

$$\left.\begin{aligned} \beta_x &= k_0 \sin(w) \quad (61) \\ k_0^2 &= \beta_x^2 + s_{z0}^2 \quad (2.13) \end{aligned}\right\} \Rightarrow \; s_{z0} = k_0 \cos(w). \tag{62}$$

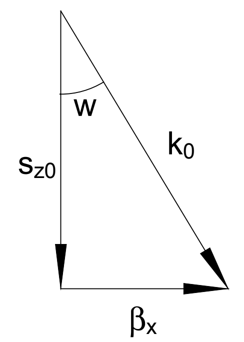


Figure 3.10: Graphical representation of the trigonometric transformation

The real part and imaginary part of the complex phase constant β_x can be written in terms of ξ and η

$$\begin{aligned} \beta_x &= k_0 \sin(w) = k_0 \sin(\xi + j\eta) \\ &= k_0\left[\sin(\xi)\cos(j\eta) + \cos(\xi)\sin(j\eta)\right] \quad [9, \text{p. } 15] \\ &= k_0\left[\sin(\xi)\cosh(\eta) + j\cos(\xi)\sinh(\eta)\right] \quad [9, \text{p. } 31] \\ &\left(= \beta_x' - j\beta_x'' = \beta_x' - j\alpha_x\right). \end{aligned} \tag{63}$$

Likewise,

$$
\begin{aligned}
s_{z0} &= k_0 \cos(w) = k_0 \cos(\xi + j\eta) \\
&= k_0\left[\cos(\xi)\cos(j\eta) - \sin(\xi)\sin(j\eta)\right] \quad \text{[9, p. 15]} \\
&= k_0\left[\cos(\xi)\cosh(\eta) - j\sin(\xi)\sinh(\eta)\right] \quad \text{[9, p. 31]} \qquad (64) \\
&\left(= \beta_{z0} - j\alpha_{z0}\right).
\end{aligned}
$$

All possible traveling wave types (backward and forward propagating) can now be mapped into a strip $-\pi \leq \xi < \pi$ of the w-plane (Fig. 3.11) [8, p. 174].

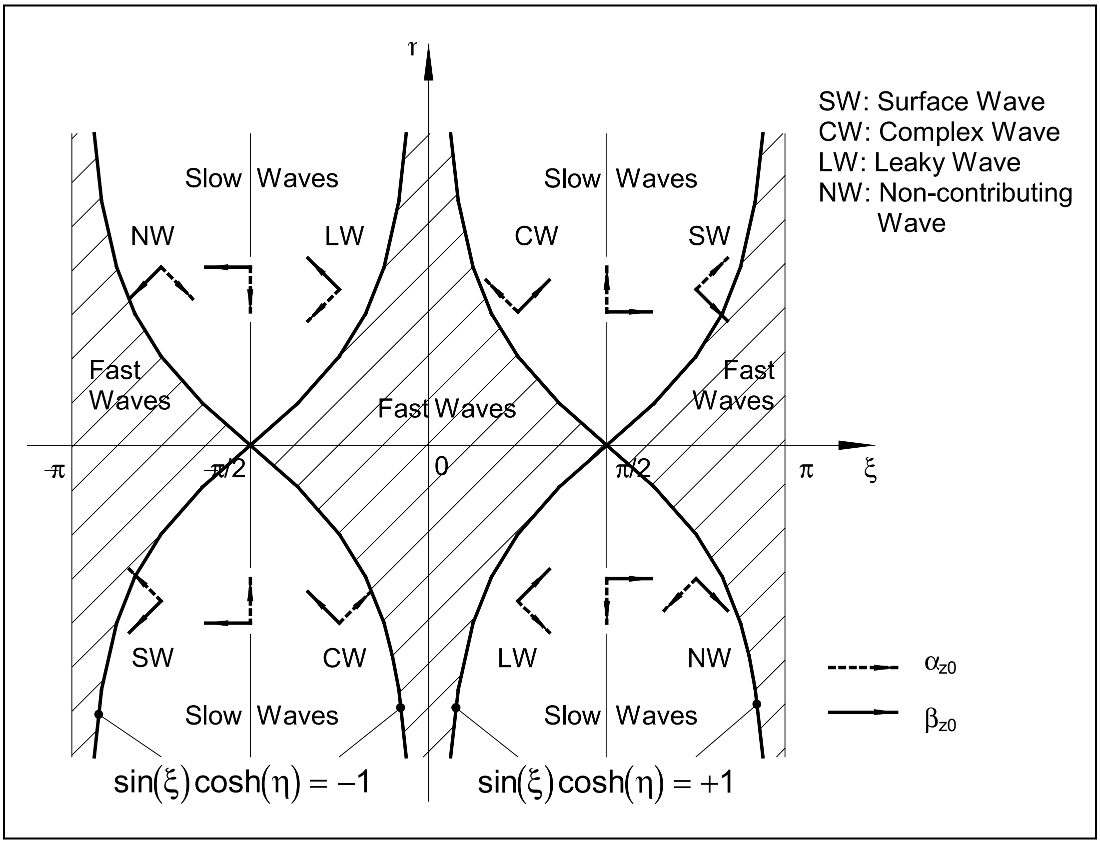


Figure 3.11: Traveling wave types as a function of eigenvalue location in the w-plane

Note that if in (64): $\eta = 0 \Rightarrow \mathrm{Im}(s_{z0}) = 0$, which corresponds to a homogeneous wave. This confirms that a complex angle of incidence implies an inhomogeneous incident wave (in fact, w could be replaced by θ).

Finally, the boundary between fast and slow waves is given by

$$v_p = \frac{\mathrm{Re}(\beta_x)}{k_0} = \sin(\xi)\cosh(\eta) = \pm 1.$$

3.4.10 Numerical Examples

Now will be shown how the discrete eigenvalue spectrum can be found graphically using Mathcad™ Plus 6.0 Professional (©1986-1995 MathSoft, Inc.). Only E-type waves are investigated. The dispersion equation of the proper E-type traveling waves (surface waves and complex waves) (13) is rewritten in a form more suitable for numerical analysis

$$F_{EP}(\beta_x) = \frac{\sqrt{k_1^2 - \beta_x^2}}{\sigma_1 + j\omega\varepsilon_1}\tan\left(h\sqrt{k_1^2 - \beta_x^2}\right) - \frac{\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right|} \cdot \frac{\sqrt{\beta_x^2 - k_2^2}}{\sigma_2 + j\omega\varepsilon_2} = 0\,. \qquad (65)$$

Similarly, for the improper traveling waves
(leaky waves and non-contributing waves)

$$F_{EI}(\beta_x) = \frac{\sqrt{k_1^2 - \beta_x^2}}{\sigma_1 + j\omega\varepsilon_1}\tan\left(h\sqrt{k_1^2 - \beta_x^2}\right) + \frac{\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right|} \cdot \frac{\sqrt{\beta_x^2 - k_2^2}}{\sigma_2 + j\omega\varepsilon_2} = 0\,. \qquad (66)$$

The eigenvalues of the forward propagating E-type traveling wave modes can be found graphically by plotting equations (65) and (66) respectively in function of the complex phase constant β_x (see Fig. 3.12). An eigenvalue on this plot is characterized by a null (black dot). It turns out that the nulls are often accompanied by sharp peaks in their immediate neighbourhood. Nulls are very localized features and can therefore easily be overlooked because computers can plot the value of a function only in a finite number of points. One way to prevent eigenvalues from being overlooked, is to zoom in on those regions of the plot where eigenvalues may be expected.

Another option is to use a numerical root finder to locate the eigenvalues. Each root finding algorithm requires one or more start values. However, when there is more than one mode propagating, it is often difficult to direct the root finder towards one particular eigenvalue. The Newton-Raphson algorithm, as implemented in the root finder of Mathcad™ is more prone to this defect than the secant algorithm for example. With the secant root finding method, an eigenvalue can be bracketed by appropriately specifying the two start values of the algorithm. Reference [10] describes this method in detail.

Another very elegant graphical solution method has been described in [3, pp. 712-716]. Unfortunately, this method is not applicable for lossy media.

The inability to annotate graphs in Mathcad™, made it necessary to provide additional information on these plots in the form of Figure 3.12.

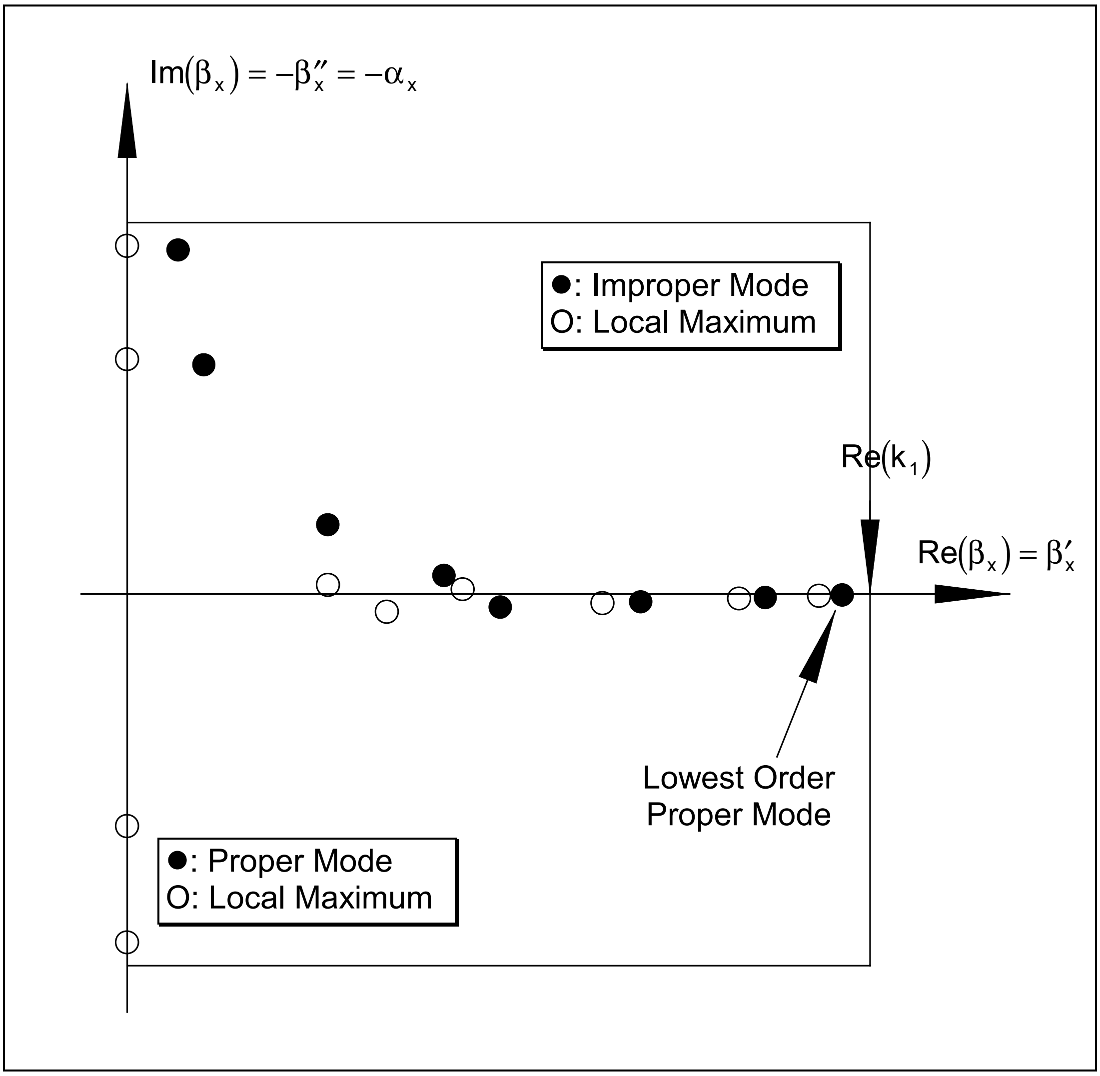


Figure 3.12: Interpretation of the complex β_x-plane plots generated by Mathcad™

Remember that is not possible to show the eigenvalues of the proper traveling wave modes and those of the improper traveling wave modes simultaneously. Improper wave modes are located in the upper half of the complex β_x-plane and proper wave modes in the lower half of the plane. Thus, for each plot produced by Mathcad™, only one half is relevant, the other half is usually a mirror image and should be ignored.

Also, for all forward propagating *lossless* surface wave modes:
$k_2 < \beta_x \le k_1$ and the lowest order surface wave mode has the highest $\text{Re}(\beta_x)$ (i.e. the slowest wave).

EXAMPLE 1
This first example shows that only one E-type surface wave mode can propagate in a plane layer of 6mm thick polyethylene (PE) on a PEC. This mode is represented by a black dot (null) on the complex β_x-plane plot. A side view of this plot is also given. (See also Fig. 3.13.)

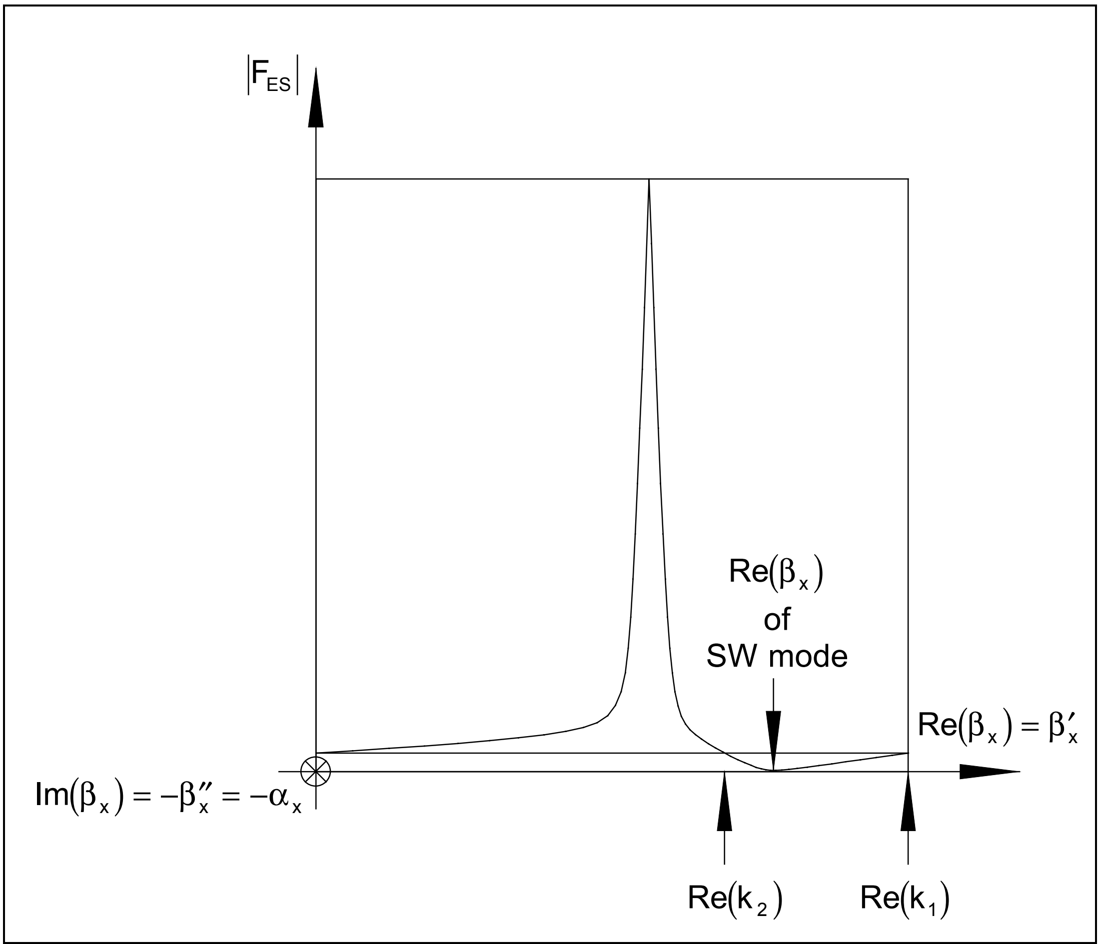


Figure 3.13: Interpretation of the side view of the complex β_x-plane plot with one E-type surface wave mode present

EXAMPLE 2
The complex β_x-plane plot of this example clearly shows that increasing the layer thickness to 15mm results in an additional proper E-type mode.

EXAMPLE 3
Further increasing the thickness to 80mm gives a multitude of proper waves. They are all surface wave modes with $\mathrm{Re}(k_2) < \mathrm{Re}(\beta_x) \leq \mathrm{Re}(k_1)$.

EXAMPLE 4
The same structure of Example 1 is now solved for improper E-type modes. Only one non-contributing wave mode is present in the upper half of the complex β_x-plane. The lower half of the plot is a mirror image and should be ignored.

EXAMPLE 5
With a coating of 15mm PE, one can discern two improper E-type modes. Ignore the lower half of the plot.

EXAMPLE 6
As the thickness of the coating further increases (here to 80mm), more and more improper E-type modes start to appear on the upper half of the plot. The lower half should be ignored.

EXAMPLE 7
This example shows the proper E-type waves along a 0.75mm thick sheet of metal-backed Eccosorb GDS, a surface wave absorbing material available from Emerson & Cuming.

EXAMPLE 8
The same configuration as in Example 7 is now solved for improper E-type modes.

EXAMPLE 9
It is also very instructive to see what happens when more losses are introduced into a relatively thick coating. From the plot can be inferred that the attenuation is higher for the higher order surface wave modes. Some of the higher order modes are fast waves and one null clearly stands out from the rest. This null corresponds to the fast surface wave that will also exist when the metal back plane is removed and the coating made infinitely thick. (See also Section 3.6.)

EXAMPLE 10
The losses in the coating are apparently that high, that no improper wave modes can be found in the upper half of the complex β_x-plane plot.

Example 1: Plane Surface Waves along a Coated PEC

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\quad \mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\quad \varepsilon_0 := \frac{1}{c_0^{\,2} \cdot \mu_0}$ $\quad \varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\quad \varepsilon_{r1} := 2.26 - 0.00091j$ $\quad \mu_{r1} := 1 - 0j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\quad \varepsilon_{r2} := 1 - 0j$ $\quad \mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 10 \cdot 10^{9} \cdot Hz$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad \omega = 6.283 \cdot 10^{10} \cdot Hz$ $\quad \lambda_0 := \frac{c_0}{f}$ $\quad \lambda_0 = 0.030 \cdot m$

Enter the thickness of the coating:

$h := 0.006 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\quad \varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\quad \mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $\quad k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot \left(\sigma_1 + j \cdot \omega \varepsilon_1\right)}$ $\quad k_1 = 315.075 - 0.063j \cdot \frac{rad}{m}$

(k$_2$ must be smaller than k$_1$!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot \left(\sigma_2 + j \cdot \omega \varepsilon_2\right)}$ $\quad k_2 = 209.585 \cdot \frac{rad}{m}$

E-type proper wave modes:

$$F_{EP}(\beta_x) := \frac{\sqrt{k_1^2-\beta_x^2}}{\sigma_1 + j\cdot\omega\varepsilon_1}\cdot\tan\left(h\cdot\sqrt{k_1^2-\beta_x^2}\right) - \frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\frac{\sqrt{\beta_x^2-k_2^2}}{\sigma_2 + j\cdot\omega\varepsilon_2}$$

$$\beta_x := \frac{k_1+k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EP}(\beta_x), \beta_x\right) \qquad \beta_x = 258.189 - 0.045j\cdot\frac{rad}{m}$$

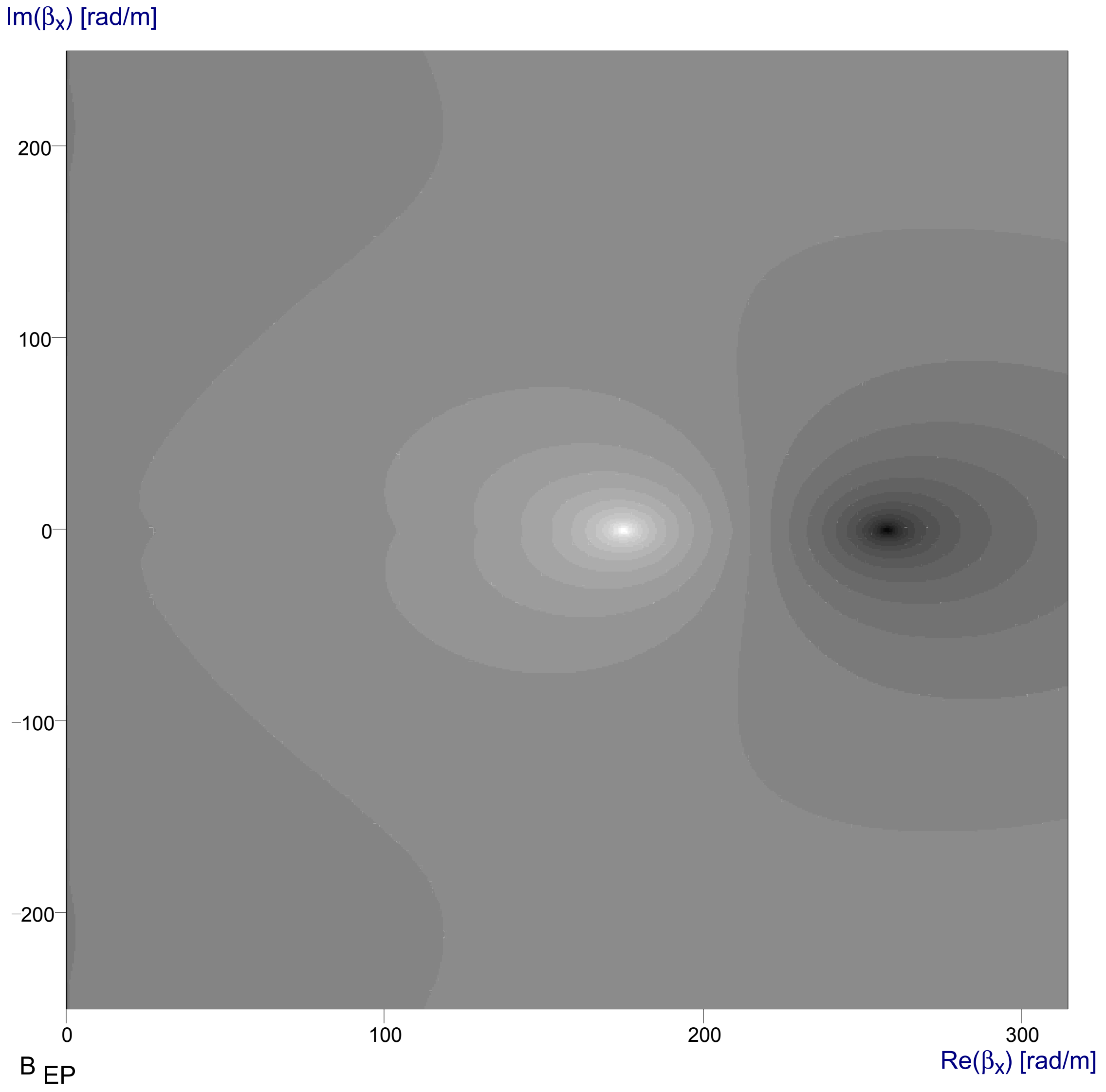
$$F_{EP}(\beta_x) = -1.392\cdot 10^{-6} + 1.134\cdot 10^{-4}j \cdot kg\cdot m^2\cdot sec^{-1}\cdot coul^{-2}$$

$$s_{z2} := -j\cdot\frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\sqrt{\beta_x^2-k_2^2} \qquad s_{z2} = -0.078 - 150.785j\cdot\frac{rad}{m}$$

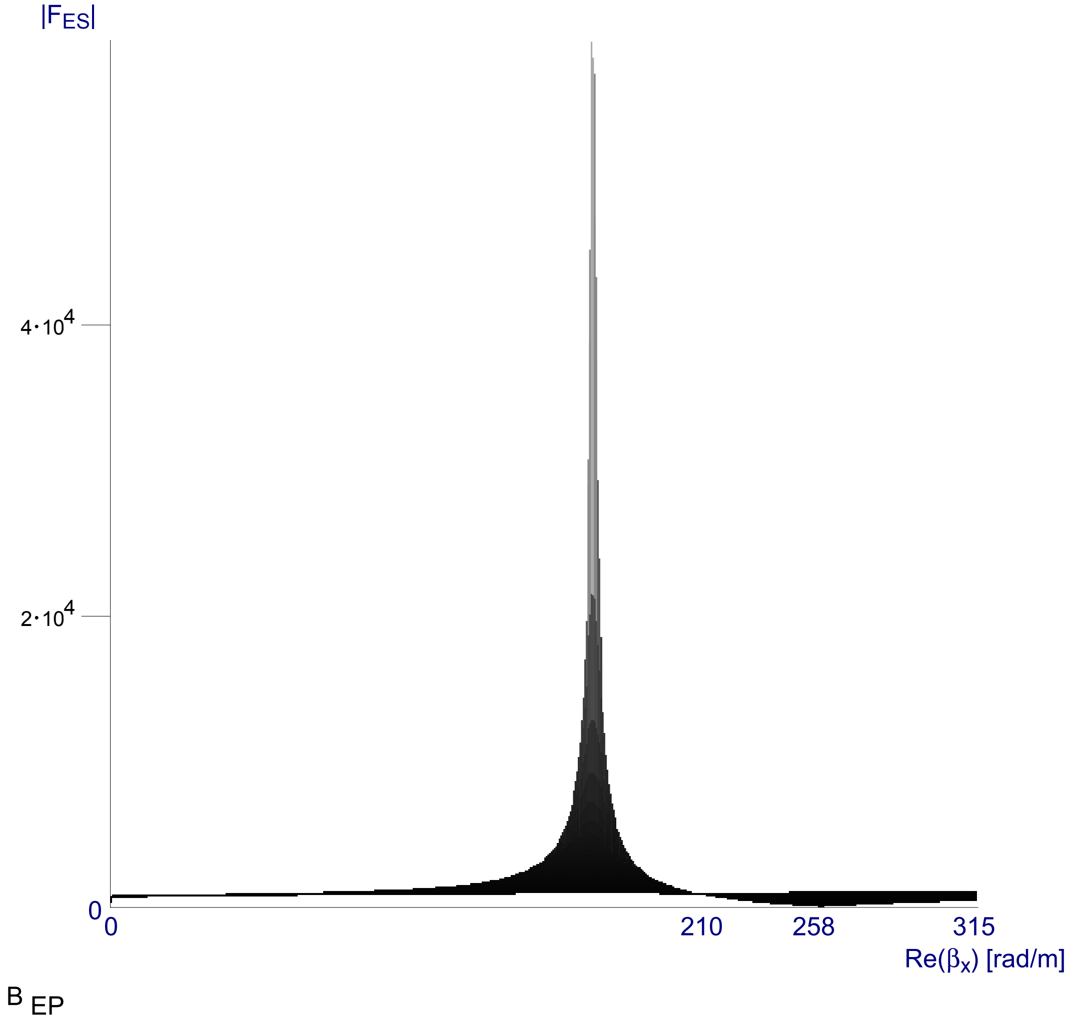
$$N := 301 \qquad Start_x := 0\cdot\frac{rad}{m} \qquad End_x := \mathrm{Re}(k_1) \qquad Start_y := -250\cdot\frac{rad}{m} \qquad End_y := 250\cdot\frac{rad}{m}$$

$$x := 0,1..N \qquad y := 0,1..N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$$

$$B_{EP_{x,y}} := \log\left[\left|F_{EP}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right|\cdot\frac{siemens}{m}\cdot\frac{m}{rad}\right]$$



$$B_{EP_{x,y}} := \left| F_{EP}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right| \cdot \frac{siemens}{m}\cdot\frac{m}{rad}$$



Example 2: Plane Surface Waves along a Coated PEC

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\quad \mu_0 := 4\cdot\pi\cdot 10^{-7} \cdot \frac{henry}{m}$ $\quad \varepsilon_0 := \frac{1}{c_0^{\,2}\cdot\mu_0}$ $\quad \varepsilon_0 = 8.854\cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0\cdot\frac{siemens}{m}$ $\quad \varepsilon_{r1} := 2.26 - 0.00091j$ $\quad \mu_{r1} := 1 - 0j$

$\sigma_2 := 0\cdot\frac{siemens}{m}$ $\quad \varepsilon_{r2} := 1 - 0j$ $\quad \mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 10\cdot 10^{9}\cdot Hz$ $\quad \omega := 2\cdot\pi\cdot f$ $\quad \omega = 6.283\cdot 10^{10} \cdot Hz$ $\quad \lambda_0 := \frac{c_0}{f}$ $\quad \lambda_0 = 0.030 \cdot m$

Enter the thickness of the coating:

$h := 0.015\cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1}\cdot\varepsilon_0$ $\quad \varepsilon_2 := \varepsilon_{r2}\cdot\varepsilon_0$

$\mu_1 := \mu_{r1}\cdot\mu_0$ $\quad \mu_2 := \mu_{r2}\cdot\mu_0$

$k_0 := \omega\sqrt{\varepsilon_0\cdot\mu_0}$ $\quad k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j\cdot\omega\mu_1\cdot(\sigma_1 + j\cdot\omega\varepsilon_1)}$ $\quad k_1 = 315.075 - 0.063j \cdot \frac{rad}{m}$

(k$_2$ must be smaller than k$_1$!)

$k_2 := \sqrt{-j\cdot\omega\mu_2\cdot(\sigma_2 + j\cdot\omega\varepsilon_2)}$ $\quad k_2 = 209.585 \cdot \frac{rad}{m}$

E-type proper wave modes:

$$F_{EP}(\beta_x) := \frac{\sqrt{k_1^2-\beta_x^2}}{\sigma_1 + j\cdot\omega\varepsilon_1}\cdot\tan\left(h\cdot\sqrt{k_1^2-\beta_x^2}\right) - \frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\frac{\sqrt{\beta_x^2-k_2^2}}{\sigma_2 + j\cdot\omega\varepsilon_2}$$

$$\beta_x := \frac{k_1+k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EP}(\beta_x),\beta_x\right) \qquad \beta_x = 212.793 - 0.02j\cdot\frac{rad}{m}$$

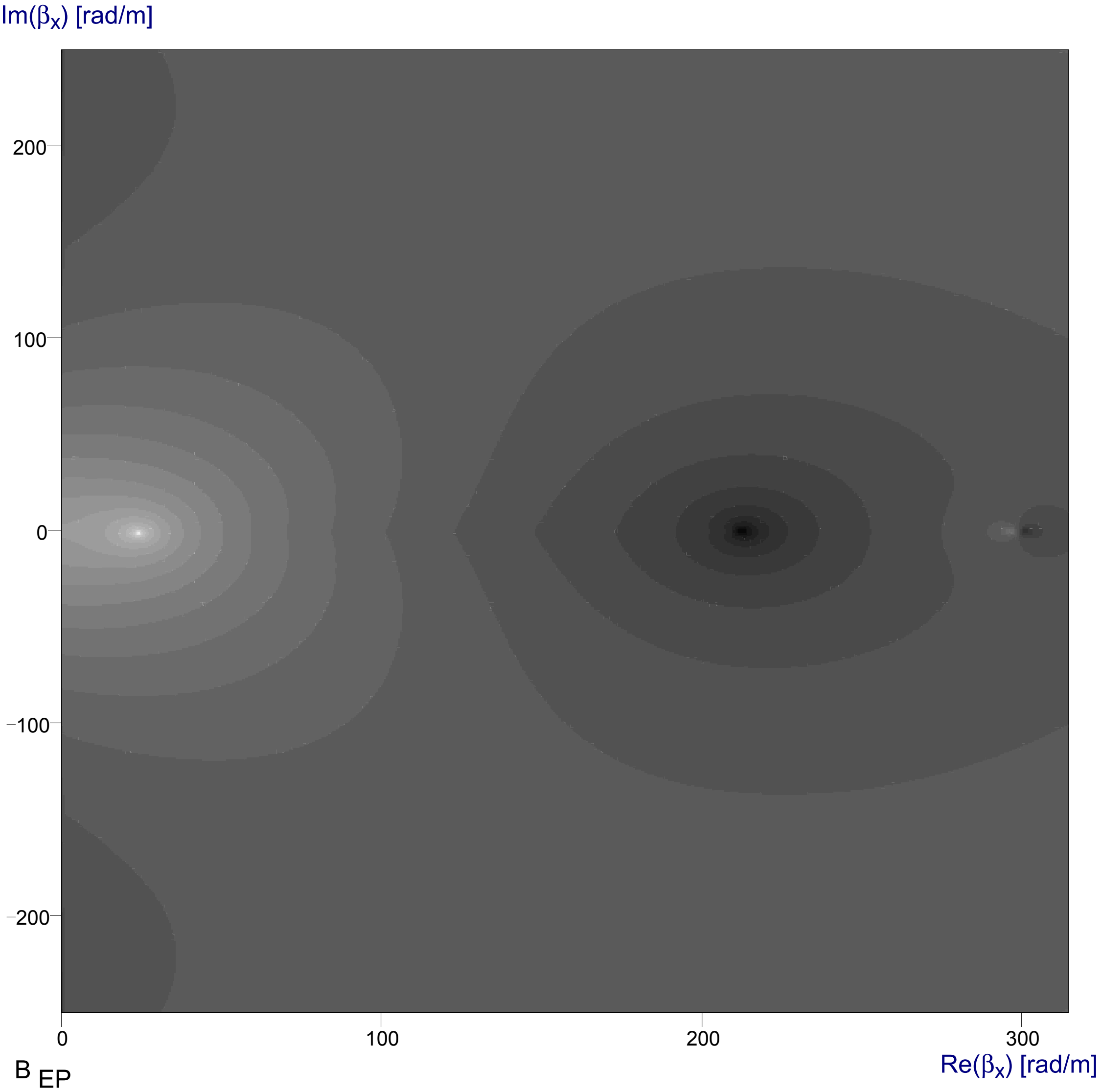
$$F_{EP}(\beta_x) = -9.521\cdot10^{-7} + 3.138\cdot10^{-7}j \cdot kg\cdot m^2\cdot sec^{-1}\cdot coul^{-2}$$

$$s_{z2} := -j\cdot\frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\sqrt{\beta_x^2-k_2^2} \qquad s_{z2} = -0.114 - 36.811j\cdot\frac{rad}{m}$$

$$N := 301 \quad Start_x := 0\cdot\frac{rad}{m} \quad End_x := \mathrm{Re}(k_1) \quad Start_y := -250\cdot\frac{rad}{m} \quad End_y := 250\cdot\frac{rad}{m}$$

$$x := 0,1..N \qquad y := 0,1..N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$$

$$B_{EP_{x,y}} := \log\left[\left|F_{EP}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right|\cdot\frac{siemens}{m}\cdot\frac{m}{rad}\right]$$



Example 3: Plane Surface Waves along a Coated PEC

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\varepsilon_0 := \frac{1}{c_0^2 \cdot \mu_0}$ $\varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r1} := 2.26 - 0.00091j$ $\mu_{r1} := 1 - 0j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r2} := 1 - 0j$ $\mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 10 \cdot 10^9 \cdot Hz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 6.283 \cdot 10^{10} \cdot Hz$ $\lambda_0 := \frac{c_0}{f}$ $\lambda_0 = 0.030 \cdot m$

Enter the thickness of the coating:

$h := 0.080 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot (\sigma_1 + j \cdot \omega \varepsilon_1)}$ $k_1 = 315.075 - 0.063j \cdot \frac{rad}{m}$

(${k_2}$ must be smaller than k_1!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot (\sigma_2 + j \cdot \omega \varepsilon_2)}$ $k_2 = 209.585 \cdot \frac{rad}{m}$

E-type proper wave modes:

$$F_{EP}(\beta_x) := \frac{\sqrt{k_1^2-\beta_x^2}}{\sigma_1 + j\cdot\omega\varepsilon_1}\cdot\tan\left(h\cdot\sqrt{k_1^2-\beta_x^2}\right) - \frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\frac{\sqrt{\beta_x^2-k_2^2}}{\sigma_2 + j\cdot\omega\varepsilon_2}$$

$\beta_x := \frac{k_1 + k_2}{2}$ $\quad\beta_x := \mathrm{root}\left(F_{EP}(\beta_x), \beta_x\right)$ $\quad\beta_x = 264.487 - 0.072j\cdot\frac{rad}{m}$

$F_{EP}(\beta_x) = -2.415\cdot10^{-4} - 4.666\cdot10^{-4}j \quad\cdot kg\cdot m^2\cdot sec^{-1}\cdot coul^{-2}$

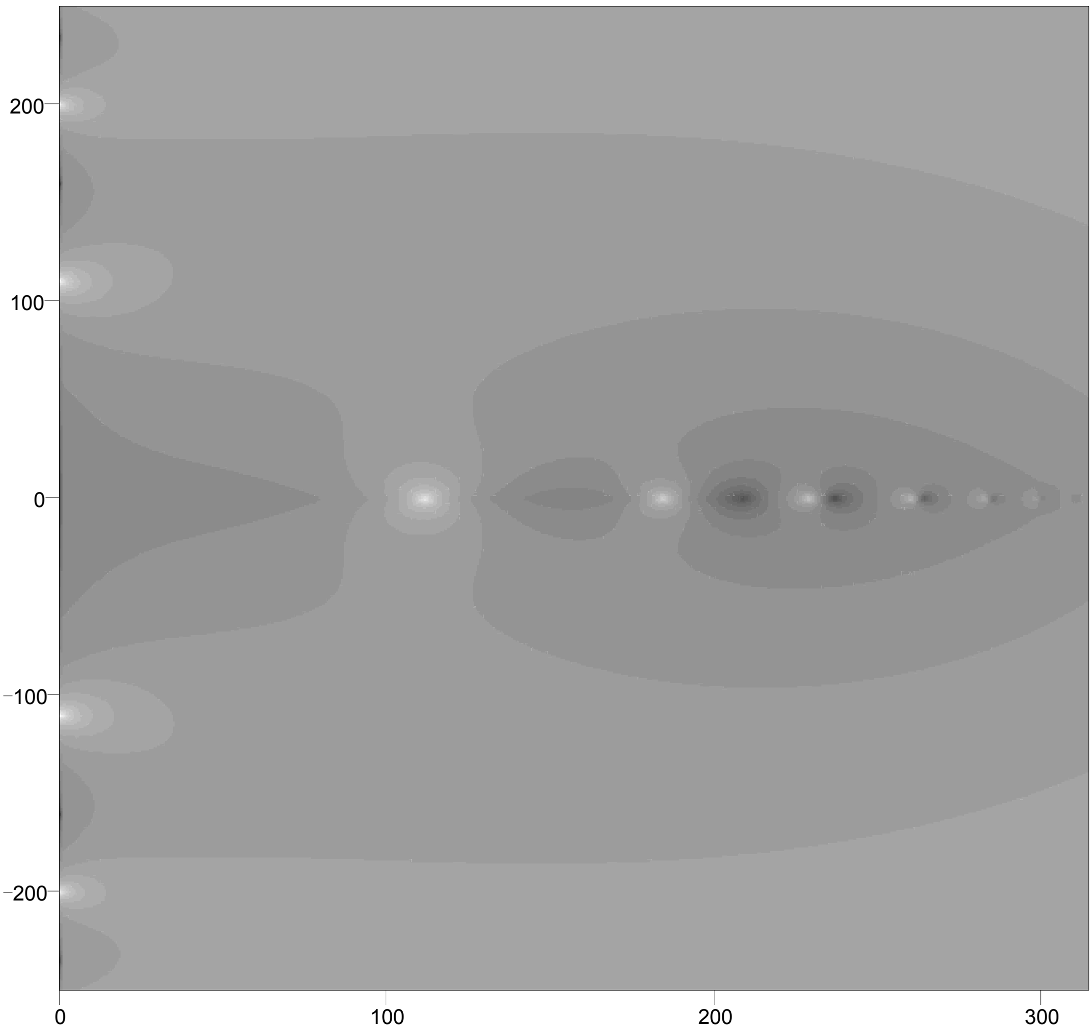
$s_{z2} := -j\cdot\frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\sqrt{\beta_x^2-k_2^2}$ $\quad s_{z2} = -0.118 - 161.331j\cdot\frac{rad}{m}$

$N := 301$ $\quad Start_x := 0\cdot\frac{rad}{m}$ $\quad End_x := \mathrm{Re}(k_1)$ $\quad Start_y := -250\cdot\frac{rad}{m}$ $\quad End_y := 250\cdot\frac{rad}{m}$

$x := 0, 1..N$ $\quad y := 0, 1..N$ $\quad \Delta x := \frac{End_x - Start_x}{N}$ $\quad \Delta y := \frac{End_y - Start_y}{N}$

$$B_{EP_{x,y}} := \log\left[\left|F_{EP}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right|\cdot\frac{siemens}{m}\cdot\frac{m}{rad}\right]$$

Im(β_x) [rad/m]



B_{EP} $\quad$ Re(β_x) [rad/m]

Example 4: Plane Improper Waves along a Coated PEC

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\varepsilon_0 := \frac{1}{c_0^2 \cdot \mu_0}$ $\varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r1} := 2.26 - 0.00091j$ $\mu_{r1} := 1 - 0j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r2} := 1 - 0j$ $\mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 10 \cdot 10^9 \cdot Hz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 6.283 \cdot 10^{10} \cdot Hz$ $\lambda_0 := \frac{c_0}{f}$ $\lambda_0 = 0.030 \cdot m$

Enter the thickness of the coating:

$h := 0.006 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot (\sigma_1 + j \cdot \omega \varepsilon_1)}$ $k_1 = 315.075 - 0.063j \cdot \frac{rad}{m}$

(k_2 must be smaller than k_1!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot (\sigma_2 + j \cdot \omega \varepsilon_2)}$ $k_2 = 209.585 \cdot \frac{rad}{m}$

E-type improper wave modes:

$$F_{EI}(\beta_x) := \frac{\sqrt{k_1^2-\beta_x^2}}{\sigma_1 + j\cdot\omega\varepsilon_1}\cdot\tan\left(h\cdot\sqrt{k_1^2-\beta_x^2}\right) + \frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\frac{\sqrt{\beta_x^2-k_2^2}}{\sigma_2 + j\cdot\omega\varepsilon_2}$$

$$\beta_x := \frac{k_1+k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EI}(\beta_x), \beta_x\right) \qquad \beta_x = 201.038 + 141.062j\cdot\frac{rad}{m}$$

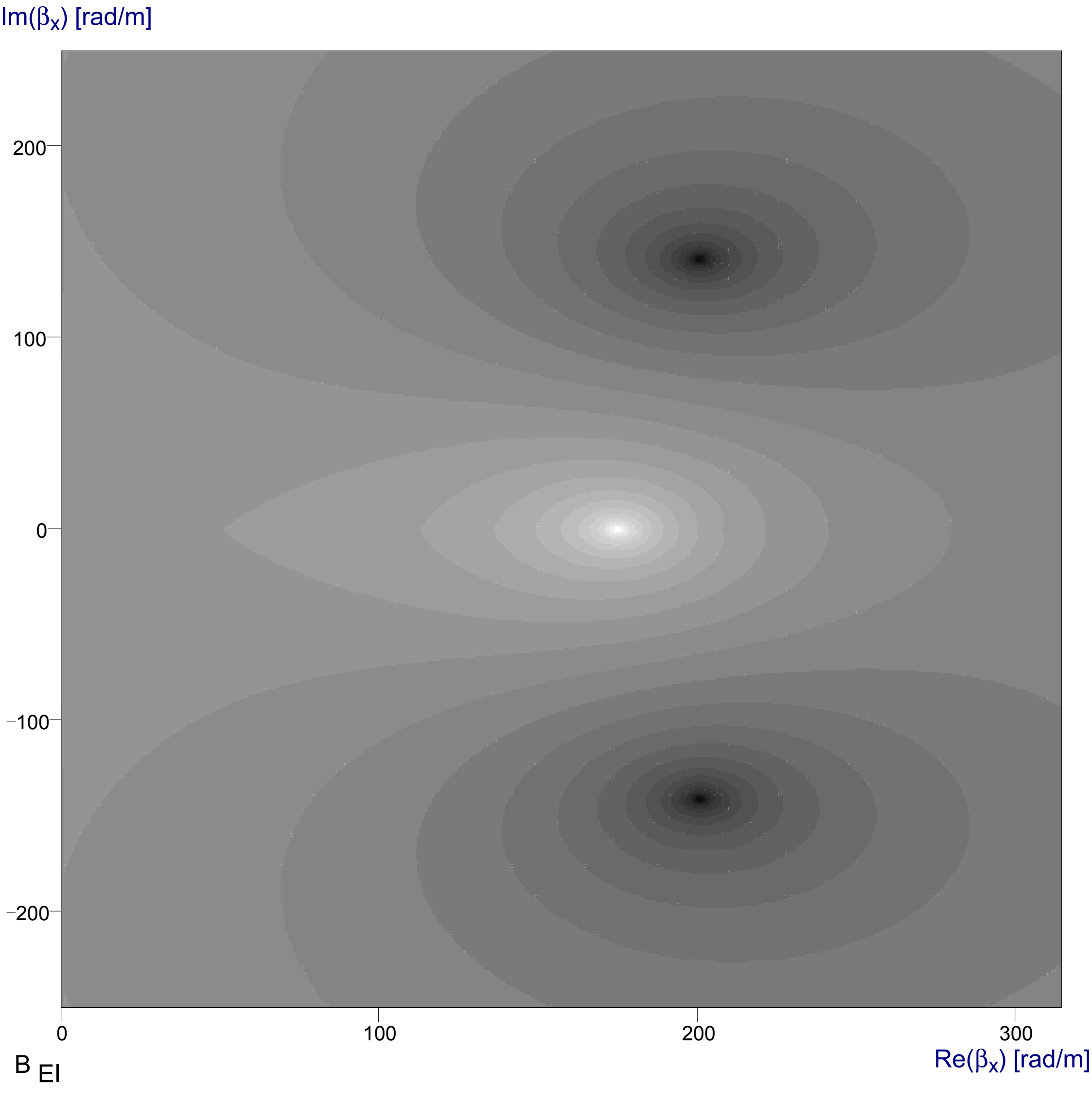
$$F_{EI}(\beta_x) = 3.234\cdot 10^{-4} + 1.567\cdot 10^{-4}j \quad \cdot kg\cdot m^2\cdot sec^{-1}\cdot coul^{-2}$$

$$s_{z2} := j\cdot\frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\sqrt{\beta_x^2-k_2^2} \qquad s_{z2} = -205.872 + 137.75j\cdot\frac{rad}{m}$$

$$N := 301 \quad Start_x := 0\cdot\frac{rad}{m} \quad End_x := \mathrm{Re}(k_1) \quad Start_y := -250\cdot\frac{rad}{m} \quad End_y := 250\cdot\frac{rad}{m}$$

$$x := 0,1..N \quad y := 0,1..N \quad \Delta x := \frac{End_x - Start_x}{N} \quad \Delta y := \frac{End_y - Start_y}{N}$$

$$B_{EI_{x,y}} := \log\left[\left|F_{EI}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right|\cdot\frac{siemens}{m}\cdot\frac{m}{rad}\right]$$



Example 5: Plane Improper Waves along a Coated PEC

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\varepsilon_0 := \frac{1}{c_0^{\,2} \cdot \mu_0}$ $\varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r1} := 2.26 - 0.00091j$ $\mu_{r1} := 1 - 0j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r2} := 1 - 0j$ $\mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 10 \cdot 10^9 \cdot Hz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 6.283 \cdot 10^{10} \cdot Hz$ $\lambda_0 := \frac{c_0}{f}$ $\lambda_0 = 0.030 \cdot m$

Enter the thickness of the coating:

$h := 0.015 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot (\sigma_1 + j \cdot \omega \varepsilon_1)}$ $k_1 = 315.075 - 0.063j \cdot \frac{rad}{m}$

(k_2 must be smaller than k_1!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot (\sigma_2 + j \cdot \omega \varepsilon_2)}$ $k_2 = 209.585 \cdot \frac{rad}{m}$

E-type improper wave modes:

$$F_{EI}(\beta_x) := \frac{\sqrt{k_1^2-\beta_x^2}}{\sigma_1 + j\cdot\omega\varepsilon_1}\cdot\tan\left(h\cdot\sqrt{k_1^2-\beta_x^2}\right) + \frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\frac{\sqrt{\beta_x^2-k_2^2}}{\sigma_2 + j\cdot\omega\varepsilon_2}$$

$$\beta_x := \frac{k_1 + k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EI}(\beta_x), \beta_x\right) \qquad \beta_x = 262.33 - 0.032j \cdot \frac{rad}{m}$$

not converging

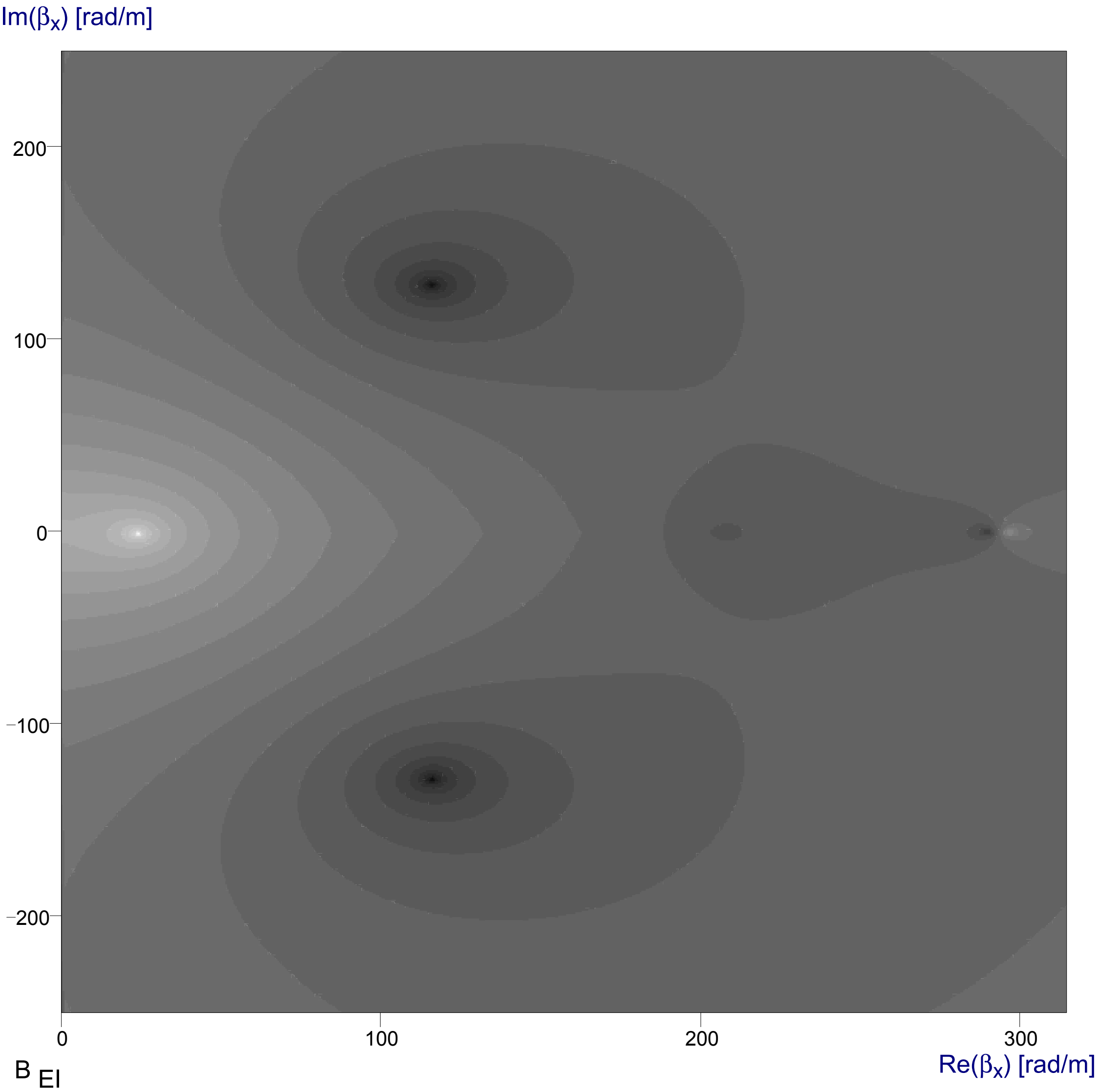
$$F_{EI}(\beta_x) = -0.282 - 203.403j \cdot kg \cdot m^2 \cdot sec^{-1} \cdot coul^{-2}$$

$$s_{z2} := j\cdot\frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\sqrt{\beta_x^2-k_2^2} \qquad s_{z2} = 0.053 + 157.769j\cdot\frac{rad}{m}$$

$$N := 301 \qquad Start_x := 0\cdot\frac{rad}{m} \qquad End_x := \mathrm{Re}(k_1) \qquad Start_y := -250\cdot\frac{rad}{m} \qquad End_y := 250\cdot\frac{rad}{m}$$

$$x := 0, 1 .. N \qquad y := 0, 1 .. N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$$

$$B_{EI_{x,y}} := \log\left[\left|F_{EI}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right|\cdot\frac{siemens}{m}\cdot\frac{m}{rad}\right]$$



Example 6: Plane Improper Waves along a Coated PEC

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\varepsilon_0 := \frac{1}{c_0^{\,2} \cdot \mu_0}$ $\varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r1} := 2.26 - 0.00091j$ $\mu_{r1} := 1 - 0j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r2} := 1 - 0j$ $\mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 10 \cdot 10^9 \cdot Hz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 6.283 \cdot 10^{10} \cdot Hz$ $\lambda_0 := \frac{c_0}{f}$ $\lambda_0 = 0.030 \cdot m$

Enter the thickness of the coating:

$h := 0.080 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot \left(\sigma_1 + j \cdot \omega \varepsilon_1\right)}$ $k_1 = 315.075 - 0.063j \cdot \frac{rad}{m}$

(k_2 must be smaller than k_1!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot \left(\sigma_2 + j \cdot \omega \varepsilon_2\right)}$ $k_2 = 209.585 \cdot \frac{rad}{m}$

E-type improper wave modes:

$$F_{EI}(\beta_x) := \frac{\sqrt{k_1^2 - \beta_x^2}}{\sigma_1 + j \cdot \omega\varepsilon_1} \cdot \tan\left(h \cdot \sqrt{k_1^2 - \beta_x^2}\right) + \frac{\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right|} \cdot \frac{\sqrt{\beta_x^2 - k_2^2}}{\sigma_2 + j \cdot \omega\varepsilon_2}$$

$$\beta_x := \frac{k_1 + k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EI}(\beta_x), \beta_x\right) \qquad \beta_x = 113.224 - 29.77j \cdot \frac{rad}{m}$$

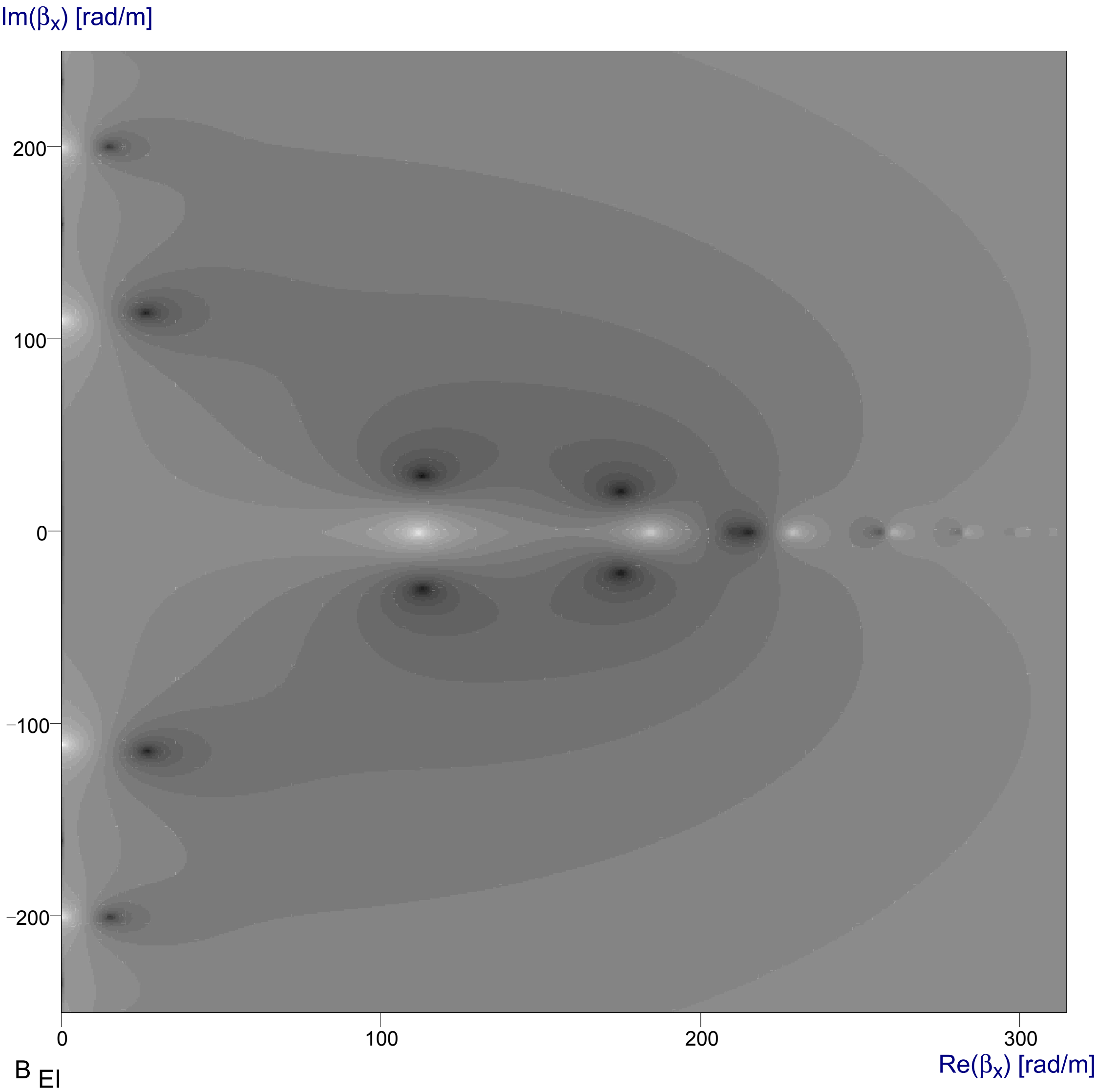
$$F_{EI}(\beta_x) = -7.483 \cdot 10^{-7} - 2.621 \cdot 10^{-6}j \quad \cdot kg \cdot m^2 \cdot sec^{-1} \cdot coul^{-2}$$

$$s_{z2} := j \cdot \frac{\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2 - k_2^2}\right)\right|} \cdot \sqrt{\beta_x^2 - k_2^2} \qquad s_{z2} = 179.843 + 18.743j \cdot \frac{rad}{m}$$

$$N := 301 \qquad Start_x := 0 \cdot \frac{rad}{m} \qquad End_x := \mathrm{Re}(k_1) \qquad Start_y := -250 \cdot \frac{rad}{m} \qquad End_y := 250 \cdot \frac{rad}{m}$$

$$x := 0, 1 .. N \qquad y := 0, 1 .. N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$$

$$B_{EI_{x,y}} := \log\left[\left|F_{EI}\left[\left(Start_x + x \cdot \Delta x\right) + \left[j \cdot \left(Start_y + y \cdot \Delta y\right)\right]\right]\right| \cdot \frac{siemens}{m} \cdot \frac{m}{rad}\right]$$



Example 7: Proper Waves along Metal-Coated Eccosorb GDS

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\varepsilon_0 := \frac{1}{c_0^2 \cdot \mu_0}$ $\varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r1} := 7.4 - 0.15j$ $\mu_{r1} := 1.4 - 0.48j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r2} := 1 - 0j$ $\mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 8.6 \cdot 10^9 \cdot Hz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 5.404 \cdot 10^{10} \cdot Hz$ $\lambda_0 := \frac{c_0}{f}$ $\lambda_0 = 0.035 \cdot m$

Enter the thickness of the coating:

$h := 0.00075 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $k_0 = 180.243 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot (\sigma_1 + j \cdot \omega \varepsilon_1)}$ $k_1 = 587.412 - 104.031j \cdot \frac{rad}{m}$

(k_2 must be smaller than k_1!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot (\sigma_2 + j \cdot \omega \varepsilon_2)}$ $k_2 = 180.243 \cdot \frac{rad}{m}$

E-type proper wave modes:

$$F_{EP}(\beta_x) := \frac{\sqrt{k_1^2-\beta_x^2}}{\sigma_1+j\cdot\omega\varepsilon_1}\cdot\tan\left(h\cdot\sqrt{k_1^2-\beta_x^2}\right) - \frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\frac{\sqrt{\beta_x^2-k_2^2}}{\sigma_2+j\cdot\omega\varepsilon_2}$$

$$\beta_x := \frac{k_1+k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EP}(\beta_x),\beta_x\right) \qquad \beta_x = 182.647 - 2.328j \cdot\frac{rad}{m}$$

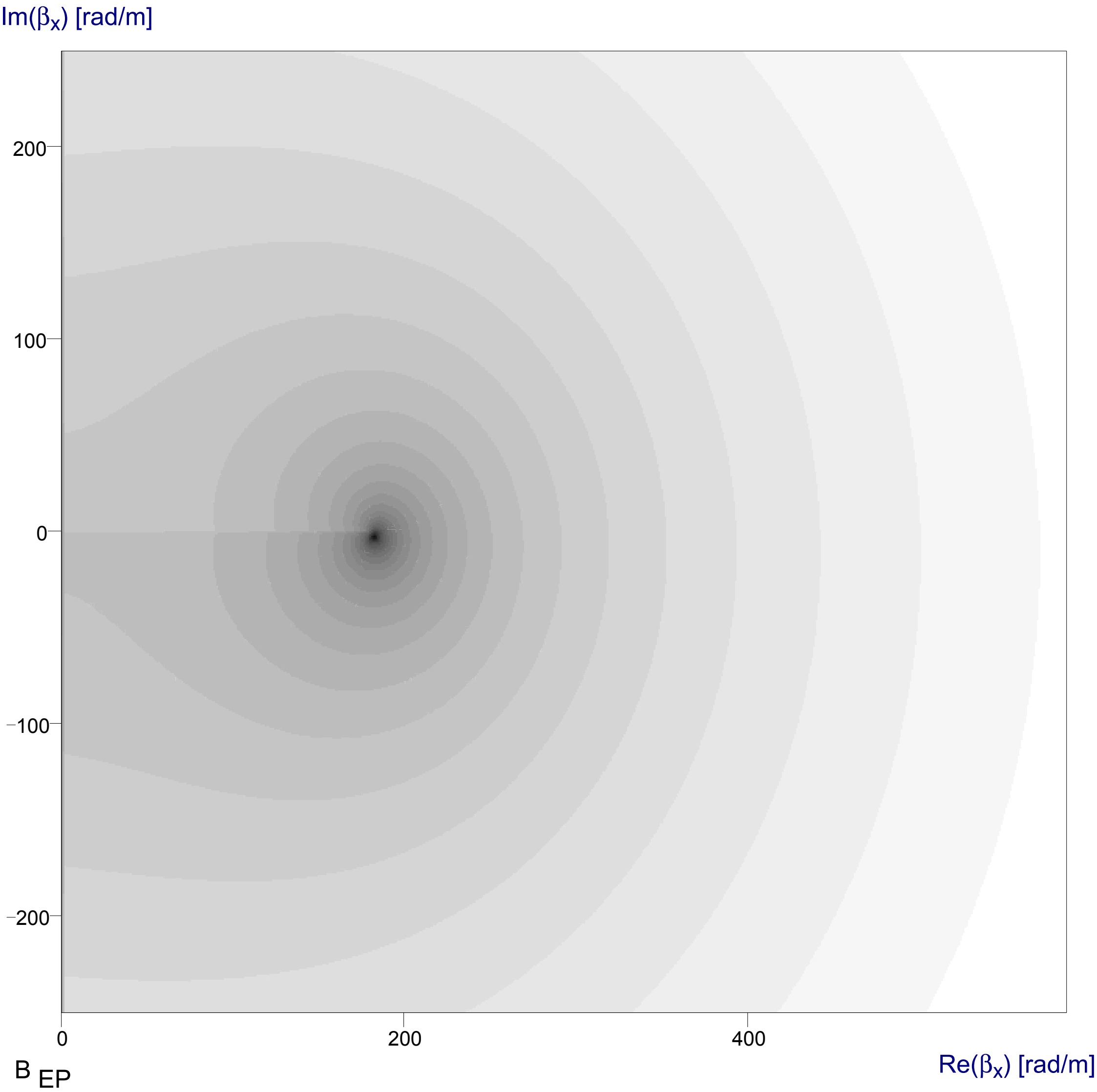
$$F_{EP}(\beta_x) = -2.203\cdot10^{-4} + 6.845\cdot10^{-4}j \cdot kg\cdot m^2\cdot sec^{-1}\cdot coul^{-2}$$

$$s_{z2} := -j\cdot\frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\sqrt{\beta_x^2-k_2^2} \qquad s_{z2} = -13.179 - 32.262j\cdot\frac{rad}{m}$$

$$N := 301 \qquad Start_x := 0\cdot\frac{rad}{m} \qquad End_x := \mathrm{Re}(k_1) \qquad Start_y := -250\cdot\frac{rad}{m} \qquad End_y := 250\cdot\frac{rad}{m}$$

$$x := 0,1..N \qquad y := 0,1..N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$$

$$B_{EP_{x,y}} := \log\left[\left|F_{EP}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right|\cdot\frac{siemens}{m}\cdot\frac{m}{rad}\right]$$



Example 8: Plane Improper Waves along Metal-Backed Eccosorb GDS

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\varepsilon_0 := \frac{1}{c_0^2 \cdot \mu_0}$ $\varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r1} := 7.4 - 0.15j$ $\mu_{r1} := 1.4 - 0.48j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r2} := 1 - 0j$ $\mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 8.6 \cdot 10^9 \cdot Hz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 5.404 \cdot 10^{10} \cdot Hz$ $\lambda_0 := \frac{c_0}{f}$ $\lambda_0 = 0.035 \cdot m$

Enter the thickness of the coating:

$h := 0.00075 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $k_0 = 180.243 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot (\sigma_1 + j \cdot \omega \varepsilon_1)}$ $k_1 = 587.412 - 104.031j \cdot \frac{rad}{m}$

(k_2 must be smaller than k_1!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot (\sigma_2 + j \cdot \omega \varepsilon_2)}$ $k_2 = 180.243 \cdot \frac{rad}{m}$

E-type improper wave modes:

$$F_{EI}(\beta_x) := \frac{\sqrt{k_1^2 - \beta_x^2}}{\sigma_1 + j\cdot\omega\varepsilon_1}\cdot\tan\left(h\cdot\sqrt{k_1^2-\beta_x^2}\right) + \frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\frac{\sqrt{\beta_x^2-k_2^2}}{\sigma_2+j\cdot\omega\varepsilon_2}$$

$$\beta_x := \frac{k_1+k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EI}(\beta_x), \beta_x\right) \qquad \beta_x = 383.827 - 52.015j\cdot\frac{\mathrm{rad}}{\mathrm{m}}$$

not converging

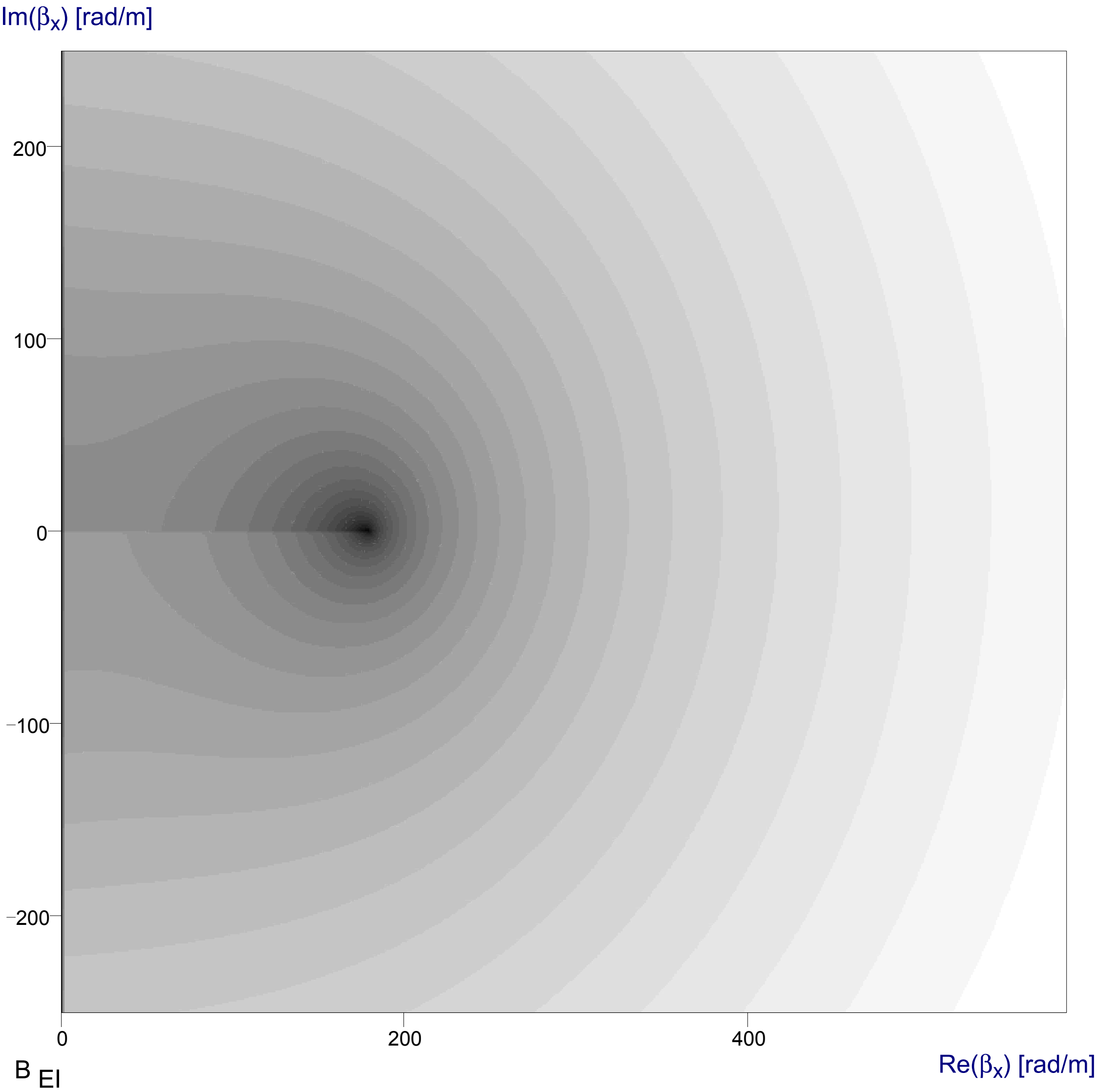
$$F_{EI}(\beta_x) = -140.648 - 752.295j\cdot\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{sec}^{-1}\cdot\mathrm{coul}^{-2}$$

$$s_{z2} := j\cdot\frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\sqrt{\beta_x^2-k_2^2} \qquad s_{z2} = 58.726 + 339.969j\cdot\frac{\mathrm{rad}}{\mathrm{m}}$$

$$N := 301 \qquad Start_x := 0\cdot\frac{\mathrm{rad}}{\mathrm{m}} \qquad End_x := \mathrm{Re}(k_1) \qquad Start_y := -250\cdot\frac{\mathrm{rad}}{\mathrm{m}} \qquad End_y := 250\cdot\frac{\mathrm{rad}}{\mathrm{m}}$$

$$x := 0,1..N \qquad y := 0,1..N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$$

$$B_{EI_{x,y}} := \log\left[\left|F_{EI}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right|\cdot\frac{\mathrm{siemens}}{\mathrm{m}}\cdot\frac{\mathrm{m}}{\mathrm{rad}}\right]$$



Example 9: Plane Proper Waves along Thick Lossy Materials

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\quad \mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\quad \varepsilon_0 := \frac{1}{c_0^2 \cdot \mu_0}$ $\quad \varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\quad \varepsilon_{r1} := 2.26 - 0.5j$ $\quad \mu_{r1} := 1 - 0j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\quad \varepsilon_{r2} := 1 - 0j$ $\quad \mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 10 \cdot 10^9 \cdot Hz$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad \omega = 6.283 \cdot 10^{10} \cdot Hz$ $\quad \lambda_0 := \frac{c_0}{f}$ $\quad \lambda_0 = 0.030 \cdot m$

Enter the thickness of the coating:

$h := 0.080 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\quad \varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\quad \mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $\quad k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot \left(\sigma_1 + j \cdot \omega \varepsilon_1\right)}$ $\quad k_1 = 316.974 - 34.645j \cdot \frac{rad}{m}$

(k_2 must be smaller than k_1!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot \left(\sigma_2 + j \cdot \omega \varepsilon_2\right)}$ $\quad k_2 = 209.585 \cdot \frac{rad}{m}$

E-type proper wave modes:

$$F_{EP}(\beta_x) := \frac{\sqrt{k_1^2-\beta_x^2}}{\sigma_1 + j\cdot\omega\varepsilon_1}\cdot\tan\left(h\cdot\sqrt{k_1^2-\beta_x^2}\right) - \frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\frac{\sqrt{\beta_x^2-k_2^2}}{\sigma_2 + j\cdot\omega\varepsilon_2}$$

$$\beta_x := \frac{k_1 + k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EP}(\beta_x), \beta_x\right) \qquad \beta_x = 175.338 - 5.359j \cdot \frac{rad}{m}$$

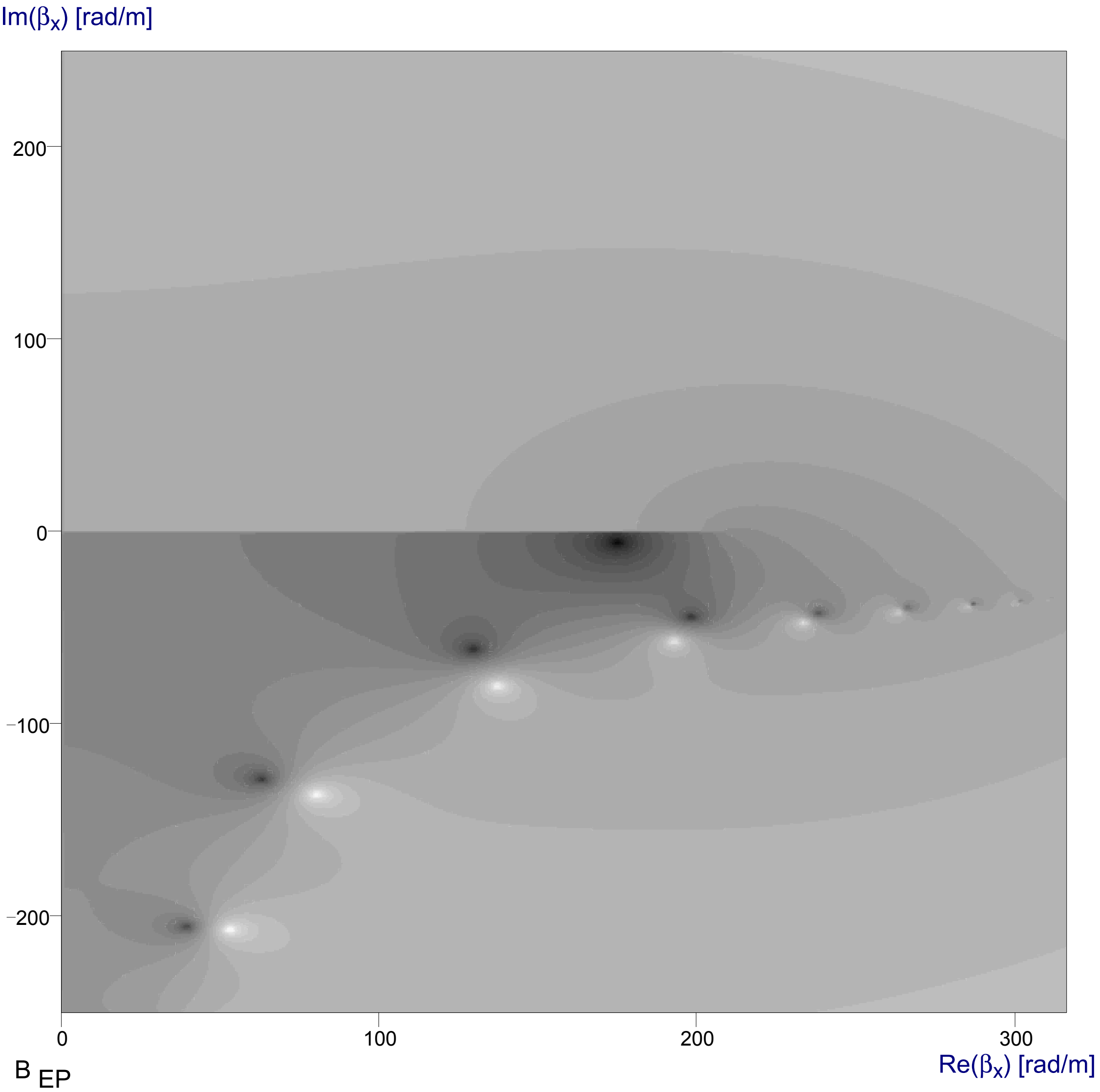
$$F_{EP}(\beta_x) = -7.218\cdot 10^{-7} + 3.616\cdot 10^{-6} j \cdot kg\cdot m^2\cdot sec^{-1}\cdot coul^{-2}$$

$$s_{z2} := -j\cdot\frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\sqrt{\beta_x^2-k_2^2} \qquad s_{z2} = -115.228 - 8.154j \cdot \frac{rad}{m}$$

$$N := 301 \qquad Start_x := 0\cdot\frac{rad}{m} \qquad End_x := \mathrm{Re}(k_1) \qquad Start_y := -250\cdot\frac{rad}{m} \qquad End_y := 250\cdot\frac{rad}{m}$$

$$x := 0, 1 .. N \qquad y := 0, 1 .. N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$$

$$B_{EP_{x,y}} := \log\left[\left|F_{EP}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right|\cdot\frac{siemens}{m}\cdot\frac{m}{rad}\right]$$



Example 10: Improper Waves along Thick Lossy Materials

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\varepsilon_0 := \frac{1}{c_0^2 \cdot \mu_0}$ $\varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r1} := 2.26 - 0.5j$ $\mu_{r1} := 1 - 0j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r2} := 1 - 0j$ $\mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 10 \cdot 10^9 \cdot Hz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 6.283 \cdot 10^{10} \cdot Hz$ $\lambda_0 := \frac{c_0}{f}$ $\lambda_0 = 0.030 \cdot m$

Enter the thickness of the coating:

$h := 0.080 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot (\sigma_1 + j \cdot \omega \varepsilon_1)}$ $k_1 = 316.974 - 34.645j \cdot \frac{rad}{m}$

(k_2 must be smaller than k_1!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot (\sigma_2 + j \cdot \omega \varepsilon_2)}$ $k_2 = 209.585 \cdot \frac{rad}{m}$

E-type improper wave modes:

$$F_{EI}(\beta_x) := \frac{\sqrt{k_1^2-\beta_x^2}}{\sigma_1 + j\cdot\omega\varepsilon_1}\cdot \tan\left(h\cdot\sqrt{k_1^2-\beta_x^2}\right) + \frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\frac{\sqrt{\beta_x^2-k_2^2}}{\sigma_2 + j\cdot\omega\varepsilon_2}$$

$$\beta_x := \frac{k_1+k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EI}(\beta_x), \beta_x\right) \qquad \beta_x = 263.279 - 17.322j\cdot\frac{\mathrm{rad}}{\mathrm{m}}$$

not converging

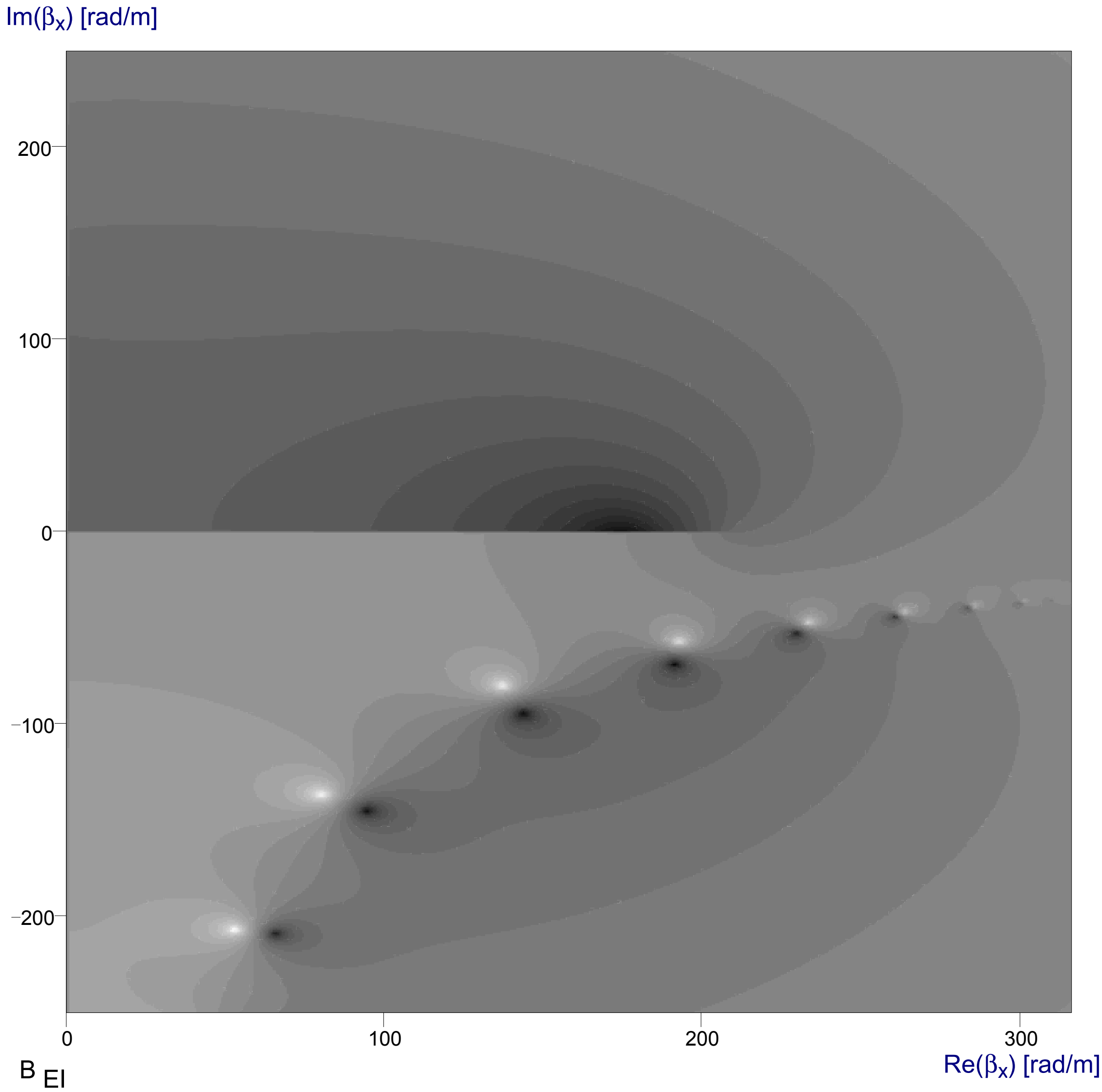
$$F_{EI}(\beta_x) = -192.578 - 291.527j\cdot \mathrm{kg}\cdot \mathrm{m}^2\cdot \mathrm{sec}^{-1}\cdot \mathrm{coul}^{-2}$$

$$s_{z2} := j\cdot\frac{\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2-k_2^2}\right)\right|}\cdot\sqrt{\beta_x^2-k_2^2} \qquad s_{z2} = 28.342 + 160.914j\cdot\frac{\mathrm{rad}}{\mathrm{m}}$$

$$N := 301 \qquad Start_x := 0\cdot\frac{\mathrm{rad}}{\mathrm{m}} \qquad End_x := \mathrm{Re}(k_1) \qquad Start_y := -250\cdot\frac{\mathrm{rad}}{\mathrm{m}} \qquad End_y := 250\cdot\frac{\mathrm{rad}}{\mathrm{m}}$$

$$x := 0, 1 .. N \qquad y := 0, 1 .. N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$$

$$B_{EI_{x,y}} := \log\left[\left|F_{EI}\left[\left(Start_x + x\cdot\Delta x\right) + \left[j\cdot\left(Start_y + y\cdot\Delta y\right)\right]\right]\right|\cdot\frac{\mathrm{siemens}}{\mathrm{m}}\cdot\frac{\mathrm{m}}{\mathrm{rad}}\right]$$



3.5 Plane Surface Waves along a Planar Three-Layer Structure

3.5.1 Introduction

The propagation of plane surface waves along the general planar three-layer topology of Figure 3.14 will be examined now. The structure consist of two half spaces with in between a plane layer of finite height h. All three media are assumed to be homogeneous, linear and isotropic.

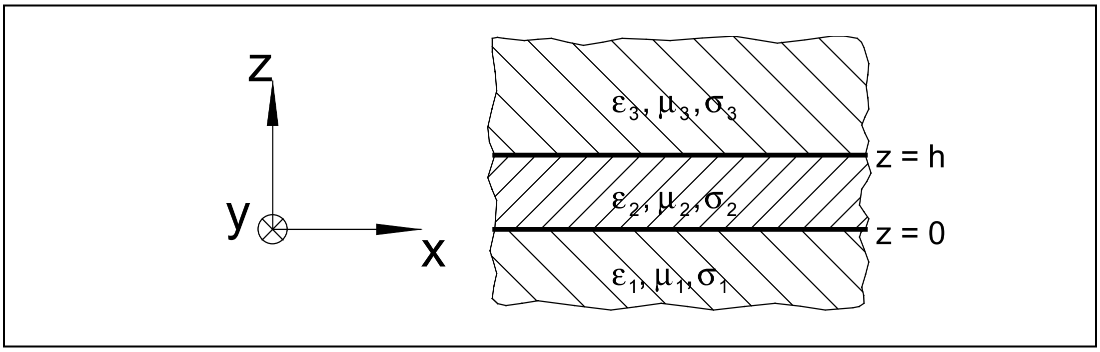


Figure 3.14: The planar three-layer structure; all three media are assumed to be homogeneous, linear and isotropic

For surface propagation it is necessary that both k_1 and k_3 are greater than k_2. These are the very same requirements to obtain a dielectric wave-guides. In fact, the waves propagating in dielectric waveguides (e.g. optical fibres) *are* surface waves. The structure of Figure 3.14 can also be used to model VLF-propagation or VHF-ducts.

3.5.2 E-Type Plane Surface Waves along a Three-Layer Structure

A suitable Hertz function for medium 1 that satisfies the boundary condition $\vec{E} = \vec{H} = \vec{0}$ when $z \to -\infty$

is $\Pi_1 = A_1 e^{+js_{z1}z} e^{-j\beta_x x}$. (67)

The factor $e^{+js_{z1}z}$ may be interpreted as a wave propagating in the negative z-direction with phase constant $s_{z1} = s'_{z1} - js''_{z1}$. The traveling wave field will show attenuation in the negative z-direction (i.e. be proper) when $s''_{z1} > 0 \;\Rightarrow\; \mathrm{Im}(s_{z1}) < 0$. If on the other hand $\mathrm{Im}(s_{z1}) > 0$, the wave will be an improper traveling wave and the radiation condition violated.

Hence, proper wave solutions are only obtained by letting

$$js_{z1} = \mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right]\sqrt{\beta_x^2 - k_1^2}\,. \tag{68a}$$

To obtain improper wave solutions, let

$$js_{z1} = -\mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right]\sqrt{\beta_x^2 - k_1^2}\,. \tag{68b}$$

Introducing (67) into (2) leads to

$$E_{z1} = A_1\left(k_1^2 - s_{z1}^2\right) e^{+js_{z1}z} e^{-j\beta_x x}, \tag{69a}$$

$$E_{x1} = \beta_x A_1 s_{z1} e^{+js_{z1}z} e^{-j\beta_x x}, \tag{69b}$$

$$E_{y1} = 0, \tag{69c}$$

$$H_{z1} = 0, \tag{69d}$$

$$H_{x1} = 0, \tag{69e}$$

$$H_{y1} = j\beta_x(\sigma_1 + j\omega\varepsilon_1) A_1 e^{+js_{z1}z} e^{-j\beta_x x}. \tag{69f}$$

A suitable Hertz function for medium 2 that can satisfy any boundary condition is

$$\Pi_2 = \left[A_{2a}\cos(s_{z2}z) + A_{2b}\sin(s_{z2}z)\right] e^{-j\beta_x x}. \tag{70}$$

In general, this corresponds to a standing wave.

Recalling (2.13)

$$s_{z2}^2 = k_2^2 - \beta_x^2 \;\Rightarrow\; s_{z2} = +\sqrt{k_2^2 - \beta_x^2}\,. \tag{71}$$

It is only for a matter of convenience that s_{z2} is chosen to equal the positive square root. Choosing the negative square root would have no effect on the results.

Introducing (70) into (2) results in

$$E_{z2} = \left(k_2^2 - s_{z2}^2\right)\left[A_{2a}\cos(s_{z2}z) + A_{2b}\sin(s_{z2}z)\right] e^{-j\beta_x x}, \tag{72a}$$

$$E_{x2} = j\beta_x s_{z2}\left[A_{2a}\sin(s_{z2}z) - A_{2b}\cos(s_{z2}z)\right] e^{-j\beta_x x}, \tag{72b}$$

$$E_{y2} = 0, \tag{72c}$$

$$H_{z2} = 0, \tag{72d}$$

$$H_{x2} = 0, \tag{72e}$$

$$H_{y2} = j\beta_x(\sigma_2 + j\omega\varepsilon_2)\left[A_{2a}\cos(s_{z2}z) + A_{2b}\sin(s_{z2}z)\right] e^{-j\beta_x x}. \tag{72f}$$

A suitable Hertz function for medium 3 that satisfies the boundary condition $\vec{E} = \vec{H} = \vec{0}$ when $z \to +\infty$

is $\Pi_3 = A_3 e^{-js_{z3}(z-h)} e^{-j\beta_x x}$. (73)

The factor $e^{-js_{z3}(z-h)}$ may be interpreted as a wave propagating in the positive z-direction with phase constant $s_{z3} = s'_{z3} - js''_{z3}$. The traveling wave field will show attenuation in the positive z-direction (i.e. be proper) when $s''_{z3} > 0 \Rightarrow \mathrm{Im}(s_{z3}) < 0$. If on the other hand $\mathrm{Im}(s_{z3}) > 0$, the wave will be an improper traveling wave and the radiation condition violated.
Hence, proper wave solutions are only obtained by letting

$$js_{z3} = \mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right]\sqrt{\beta_x^2 - k_3^2}\,. \tag{74a}$$

To obtain improper wave solutions, let

$$js_{z3} = -\mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right]\sqrt{\beta_x^2 - k_3^2}\,. \tag{74b}$$

Introducing (73) into (2) leads to

$$E_{z3} = A_3\left(k_3^2 - s_{z3}^2\right)e^{-js_{z3}(z-h)} e^{-j\beta_x x}, \tag{75a}$$

$$E_{x3} = -\beta_x A_3 s_{z3} e^{-js_{z3}(z-h)} e^{-j\beta_x x}, \tag{75b}$$

$$E_{y3} = 0, \tag{75c}$$

$$H_{z3} = 0, \tag{75d}$$

$$H_{x3} = 0, \tag{75e}$$

$$H_{y3} = j\beta_x(\sigma_3 + j\omega\varepsilon_3)A_3 e^{-js_{z3}(z-h)} e^{-j\beta_x x}. \tag{75f}$$

The tangential components of both $\vec{E}$ and $\vec{H}$ are continuous across the interface of two media and therefore

$E_{x1} = E_{x2}$ at $z = 0$

$$\Rightarrow\ A_1 s_{z1} = -jA_{2b} s_{z2}, \tag{76}$$

as well as $H_{y1} = H_{y2}$ at $z = 0$

$$\Rightarrow\ (\sigma_1 + j\omega\varepsilon_1)A_1 = (\sigma_2 + j\omega\varepsilon_2)A_{2a}. \tag{77}$$

$E_{x2} = E_{x3}$ at $z = h$

$$\Rightarrow\ js_{z2}\left[A_{2b}\cos(s_{z2}h) - A_{2a}\sin(s_{z2}h)\right] = A_3 s_{z3}, \tag{78}$$

as well as $H_{y2} = H_{y3}$ at $z = h$

$$\Rightarrow\ (\sigma_2 + j\omega\varepsilon_2)\left[A_{2a}\cos(s_{z2}h) + A_{2b}\sin(s_{z2}h)\right] = (\sigma_3 + j\omega\varepsilon_3)A_3. \tag{79}$$

Rewriting (77)

$$A_1 = \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_1 + j\omega\varepsilon_1} A_{2a}$$

and substituting the result into (76) gives

$$\frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_1 + j\omega\varepsilon_1} s_{z1} A_{2a} + j s_{z2} A_{2b} = 0\,. \tag{80}$$

Rewriting (79)

$$A_3 = \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3}\left[A_{2a}\cos(s_{z2}h) + A_{2b}\sin(s_{z2}h)\right]$$

and substituting the result into (78) leads to

$$\begin{aligned} &-\left[j s_{z2}\sin(s_{z2}h) + \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3} s_{z3}\cos(s_{z2}h)\right] A_{2a} \\ &+\left[j s_{z2}\cos(s_{z2}h) - \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3} s_{z3}\sin(s_{z2}h)\right] A_{2b} = 0\,. \end{aligned} \tag{81}$$

Equations (80) and (81) form a system of linear equations for the two unknown factors A_{2a} and A_{2b}. The system is homogeneous, hence for non-trivial solutions to exist, the coefficient determinant must be zero, that is

$$\begin{aligned} &\frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_1 + j\omega\varepsilon_1} s_{z1}\left[j s_{z2}\cos(s_{z2}h) - \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3} s_{z3}\sin(s_{z2}h)\right] \\ &+ j s_{z2}\left[j s_{z2}\sin(s_{z2}h) + \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3} s_{z3}\cos(s_{z2}h)\right] = 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow\ &\frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_1 + j\omega\varepsilon_1} s_{z1}\left[j s_{z2} - \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3} s_{z3}\tan(s_{z2}h)\right] \\ &+ j s_{z2}\left[j s_{z2}\tan(s_{z2}h) + \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3} s_{z3}\right] = 0 \end{aligned}$$

$$\Rightarrow\ \left(\frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_1 + j\omega\varepsilon_1}\cdot\frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3} s_{z1}s_{z3} + s_{z2}^2\right)\tan(s_{z2}h) = j s_{z2}\left(\frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_1 + j\omega\varepsilon_1} s_{z1} + \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3} s_{z3}\right). \tag{82}$$

Substituting (68a), (71) and (74a) into (82) results in the following expression for proper E-type traveling wave modes

$$\begin{aligned} &\left[-\frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_1 + j\omega\varepsilon_1}\cdot\frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3}\cdot \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right]\cdot\sqrt{\beta_x^2 - k_1^2}\cdot \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right]\cdot\sqrt{\beta_x^2 - k_3^2} + k_2^2 - \beta_x^2\right] \\ &\cdot\tan\left(h\sqrt{k_2^2 - \beta_x^2}\right) = \\ &\sqrt{k_2^2 - \beta_x^2}\left[\frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_1 + j\omega\varepsilon_1}\cdot \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right]\cdot\sqrt{\beta_x^2 - k_1^2} + \frac{\sigma_2 + j\omega\varepsilon_2}{\sigma_3 + j\omega\varepsilon_3}\cdot \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right]\cdot\sqrt{\beta_x^2 - k_3^2}\right]. \end{aligned} \tag{83}$$

The nomenclature of symmetrical E-type modes (i.e. when $k_1 = k_3$) is discussed in [3, pp. 712-716].

3.5.3 H-Type Plane Surface Waves along a Three-Layer Structure

A suitable Hertz function for medium 1 that satisfies the boundary condition $\vec{E} = \vec{H} = \vec{0}$ when $z \to -\infty$
is $\Pi_1 = A_1 e^{+js_{z1}z} e^{-j\beta_x x}$. (84)

For s_{z1}, the same reasoning applies as in the previous section.
Hence, proper wave solutions are obtained by letting $\mathrm{Re}(js_{z1}) \geq 0$ or

$$js_{z1} = \mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right]\sqrt{\beta_x^2 - k_1^2}\ . \tag{85a}$$

To obtain improper wave solutions, let

$$js_{z1} = -\mathrm{sign}\left[\mathrm{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right]\sqrt{\beta_x^2 - k_1^2}\ . \tag{85b}$$

Introducing (84) into (3) leads to

$$H_{z1} = A_1\left(k_1^2 - s_{z1}^2\right)e^{+js_{z1}z}e^{-j\beta_x x}, \tag{86a}$$

$$H_{x1} = \beta_x A_1 s_{z1} e^{+js_{z1}z}e^{-j\beta_x x}, \tag{86b}$$

$$H_{y1} = 0, \tag{86c}$$

$$E_{z1} = 0, \tag{86d}$$

$$E_{x1} = 0, \tag{86e}$$

$$E_{y1} = \beta_x \omega\mu_1 A_1 e^{+js_{z1}z}e^{-j\beta_x x}. \tag{86f}$$

A suitable Hertz function for medium 2 that can satisfy any boundary condition is

$$\Pi_2 = \left[A_{2a}\cos(s_{z2}z) + A_{2b}\sin(s_{z2}z)\right]e^{-j\beta_x x}. \tag{87}$$

In general, this corresponds to a standing wave.

Recalling (2.13)

$$s_{z2}^2 = k_2^2 - \beta_x^2 \ \Rightarrow\ s_{z2} = +\sqrt{k_2^2 - \beta_x^2}\ . \tag{88}$$

It is only for a matter of convenience that s_{z2} is chosen to equal the positive square root. Choosing the negative square root would have no effect on the results.

Introducing (87) into (2) results in

$$H_{z2} = \left(k_2^2 - s_{z2}^2\right)\left[A_{2a}\cos(s_{z2}z) + A_{2b}\sin(s_{z2}z)\right]e^{-j\beta_x x}, \tag{89a}$$

$$H_{x2} = j\beta_x s_{z2}\left[A_{2a}\sin(s_{z2}z) - A_{2b}\cos(s_{z2}z)\right]e^{-j\beta_x x}, \tag{89b}$$

$$H_{y2} = 0, \tag{89c}$$

$$E_{z2} = 0, \tag{89d}$$

$$E_{x2} = 0, \tag{89e}$$

$$E_{y2} = \beta_x \omega\mu_2\left[A_{2a}\cos(s_{z2}z) + A_{2b}\sin(s_{z2}z)\right]e^{-j\beta_x x}. \tag{89f}$$

A suitable Hertz function for medium 3 that satisfies the boundary condition $\vec{E} = \vec{H} = \vec{0}$ when $z \to +\infty$

is $\Pi_3 = A_3 e^{-js_{z3}(z-h)} e^{-j\beta_x x}$. (90)

For s_{z3}, the same reasoning applies as in the previous section.
Hence, proper wave solutions are obtained by letting $\text{Re}(js_{z3}) \geq 0$ or

$$js_{z3} = \text{sign}\left[\text{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right]\sqrt{\beta_x^2 - k_3^2}\,. \tag{91a}$$

To obtain improper wave solutions, let

$$js_{z3} = -\text{sign}\left[\text{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right]\sqrt{\beta_x^2 - k_3^2}\,. \tag{91b}$$

Introducing (90) into (3) leads to

$$H_{z3} = A_3\left(k_3^2 - s_{z3}^2\right)e^{-js_{z3}(z-h)}e^{-j\beta_x x}, \tag{92a}$$

$$H_{x3} = -\beta_x A_3 s_{z3} e^{-js_{z3}(z-h)}e^{-j\beta_x x}, \tag{92b}$$

$$H_{y3} = 0, \tag{92c}$$

$$E_{z3} = 0, \tag{92d}$$

$$E_{x3} = 0, \tag{92e}$$

$$E_{y3} = \beta_x \omega \mu_3 A_3 e^{-js_{z3}(z-h)}e^{-j\beta_x x}. \tag{92f}$$

The tangential components of both $\vec{E}$ and $\vec{H}$ are continuous across the interface of two media and therefore

$H_{x1} = H_{x2}$ at $z = 0$

$$\Rightarrow \; A_1 s_{z1} = -jA_{2b} s_{z2}, \tag{93}$$

as well as $E_{y1} = E_{y2}$ at $z = 0$

$$\Rightarrow \; \mu_1 A_1 = \mu_2 A_{2a}. \tag{94}$$

$H_{x2} = H_{x3}$ at $z = h$

$$\Rightarrow \; js_{z2}\left[A_{2b}\cos(s_{z2}h) - A_{2a}\sin(s_{z2}h)\right] = A_3 s_{z3}, \tag{95}$$

as well as $E_{y2} = E_{y3}$ at $z = h$

$$\Rightarrow \; \mu_2\left[A_{2a}\cos(s_{z2}h) + A_{2b}\sin(s_{z2}h)\right] = \mu_3 A_3. \tag{96}$$

Rewriting (94)

$$A_1 = \frac{\mu_2}{\mu_1} A_{2a}$$

and substituting the result into (93) gives

$$\frac{\mu_2}{\mu_1} s_{z1} A_{2a} + j s_{z2} A_{2b} = 0 . \tag{97}$$

Rewriting (96)

$$A_3 = \frac{\mu_2}{\mu_3}\left[A_{2a}\cos(s_{z2}h) + A_{2b}\sin(s_{z2}h)\right]$$

and substituting the result into (95) leads to

$$-\left[j s_{z2}\sin(s_{z2}h) + \frac{\mu_2}{\mu_3} s_{z3}\cos(s_{z2}h)\right] A_{2a}$$
$$+\left[j s_{z2}\cos(s_{z2}h) - \frac{\mu_2}{\mu_3} s_{z3}\sin(s_{z2}h)\right] A_{2b} = 0 . \tag{98}$$

Equations (97) and (98) form a system of linear equations for the two unknown factors A_{2a} and A_{2b}. The system is homogeneous, hence for non-trivial solutions to exist, the coefficient determinant must be zero, that is

$$\frac{\mu_2}{\mu_1} s_{z1}\left[j s_{z2}\cos(s_{z2}h) - \frac{\mu_2}{\mu_3} s_{z3}\sin(s_{z2}h)\right]$$
$$+ j s_{z2}\left[j s_{z2}\sin(s_{z2}h) + \frac{\mu_2}{\mu_3} s_{z3}\cos(s_{z2}h)\right] = 0$$

$$\Rightarrow \frac{\mu_2}{\mu_1} s_{z1}\left[j s_{z2} - \frac{\mu_2}{\mu_3} s_{z3}\tan(s_{z2}h)\right] + j s_{z2}\left[j s_{z2}\tan(s_{z2}h) + \frac{\mu_2}{\mu_3} s_{z3}\right] = 0$$

$$\Rightarrow \left(\frac{\mu_2}{\mu_1}\cdot\frac{\mu_2}{\mu_3} s_{z1} s_{z3} + s_{z2}^2\right)\tan(s_{z2}h) = j s_{z2}\left(\frac{\mu_2}{\mu_1} s_{z1} + \frac{\mu_2}{\mu_3} s_{z3}\right), \tag{99}$$

Substituting (85a), (88) and (91a) into (99) results in the following expression for proper H-type traveling wave modes

$$\left[-\frac{\mu_2}{\mu_1}\cdot\frac{\mu_2}{\mu_3}\cdot \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right]\cdot\sqrt{\beta_x^2 - k_1^2}\cdot \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right]\cdot\sqrt{\beta_x^2 - k_3^2} + k_2^2 - \beta_x^2\right]$$
$$\cdot\tan\left(h\sqrt{k_2^2 - \beta_x^2}\right) =$$
$$\sqrt{k_2^2 - \beta_x^2}\left[\frac{\mu_2}{\mu_1}\cdot \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right]\cdot\sqrt{\beta_x^2 - k_1^2} + \frac{\mu_2}{\mu_3}\cdot \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right]\cdot\sqrt{\beta_x^2 - k_3^2}\right]. \tag{100}$$

This equation is transcendental and can therefore only be solved numerically for β_x.

H-type surface wave modes in a three-layer structure can be subdivided into odd and even modes in the special case when k_1 equals k_3. These symmetrical H-type modes are further discussed in [3, pp. 712-716].

Example: Plane Proper Waves along a Dielectric Waveguide

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\varepsilon_0 := \frac{1}{c_0^2 \cdot \mu_0}$ $\varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r1} := 1 - 0j$ $\mu_{r1} := 1 - 0j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r2} := 2.26 - 0.00091j$ $\mu_{r2} := 1 - 0j$

$\sigma_3 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r3} := 1 - 0j$ $\mu_{r3} := 1 - 0j$

Enter the frequency:

$f := 10 \cdot 10^9 \cdot Hz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 6.283 \cdot 10^{10} \cdot Hz$ $\lambda_0 := \frac{c_0}{f}$ $\lambda_0 = 0.030 \cdot m$

Enter the thickness of the middle layer:

$h := 0.006 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$ $\varepsilon_3 := \varepsilon_{r3} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\mu_2 := \mu_{r2} \cdot \mu_0$ $\mu_3 := \mu_{r3} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot (\sigma_1 + j \cdot \omega \varepsilon_1)}$ $k_1 = 209.585 \cdot \frac{rad}{m}$ (k_1 must be smaller than k_2!)

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot (\sigma_2 + j \cdot \omega \varepsilon_2)}$ $k_2 = 315.075 - 0.063j \cdot \frac{rad}{m}$

$k_3 := \sqrt{-j \cdot \omega \mu_3 \cdot (\sigma_3 + j \cdot \omega \varepsilon_3)}$ $k_3 = 209.585 \cdot \frac{rad}{m}$ (k_3 must be smaller than k_2!)

E-type proper wave modes:

$$js_{z1}(\beta_x) := \frac{\mathrm{Re}\left(\sqrt{\beta_x^{\,2} - k_1^{\,2}}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^{\,2} - k_1^{\,2}}\right)\right|} \cdot \sqrt{\beta_x^{\,2} - k_1^{\,2}}$$

$$js_{z3}(\beta_x) := \frac{\mathrm{Re}\left(\sqrt{\beta_x^{\,2} - k_3^{\,2}}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^{\,2} - k_3^{\,2}}\right)\right|} \cdot \sqrt{\beta_x^{\,2} - k_3^{\,2}}$$

$$F_{EP1}(\beta_x) := \left(-\frac{\sigma_2 + j \cdot \omega\varepsilon_2}{\sigma_1 + j \cdot \omega\varepsilon_1} \cdot \frac{\sigma_2 + j \cdot \omega\varepsilon_2}{\sigma_3 + j \cdot \omega\varepsilon_3} \cdot js_{z1}(\beta_x) \cdot js_{z3}(\beta_x) + k_2^{\,2} - \beta_x^{\,2}\right)$$

$$F_{EP2}(\beta_x) := \tan\left(h \cdot \sqrt{k_2^{\,2} - \beta_x^{\,2}}\right)$$

$$F_{EP3}(\beta_x) := \sqrt{k_2^{\,2} - \beta_x^{\,2}} \cdot \left(\frac{\sigma_2 + j \cdot \omega\varepsilon_2}{\sigma_1 + j \cdot \omega\varepsilon_1} \cdot js_{z1}(\beta_x) + \frac{\sigma_2 + j \cdot \omega\varepsilon_2}{\sigma_3 + j \cdot \omega\varepsilon_3} \cdot js_{z3}(\beta_x)\right)$$

$$F_{EP}(\beta_x) := F_{EP1}(\beta_x) \cdot F_{EP2}(\beta_x) - F_{EP3}(\beta_x)$$

$\beta_x := k_2$ $\quad$ $\beta_x := \mathrm{root}\left(F_{EP}(\beta_x), \beta_x\right)$ $\quad$ $\beta_x = 315.075 - 0.063j \cdot \frac{rad}{m}$

$F_{EP}(\beta_x) = 0 \cdot m^{-2}$

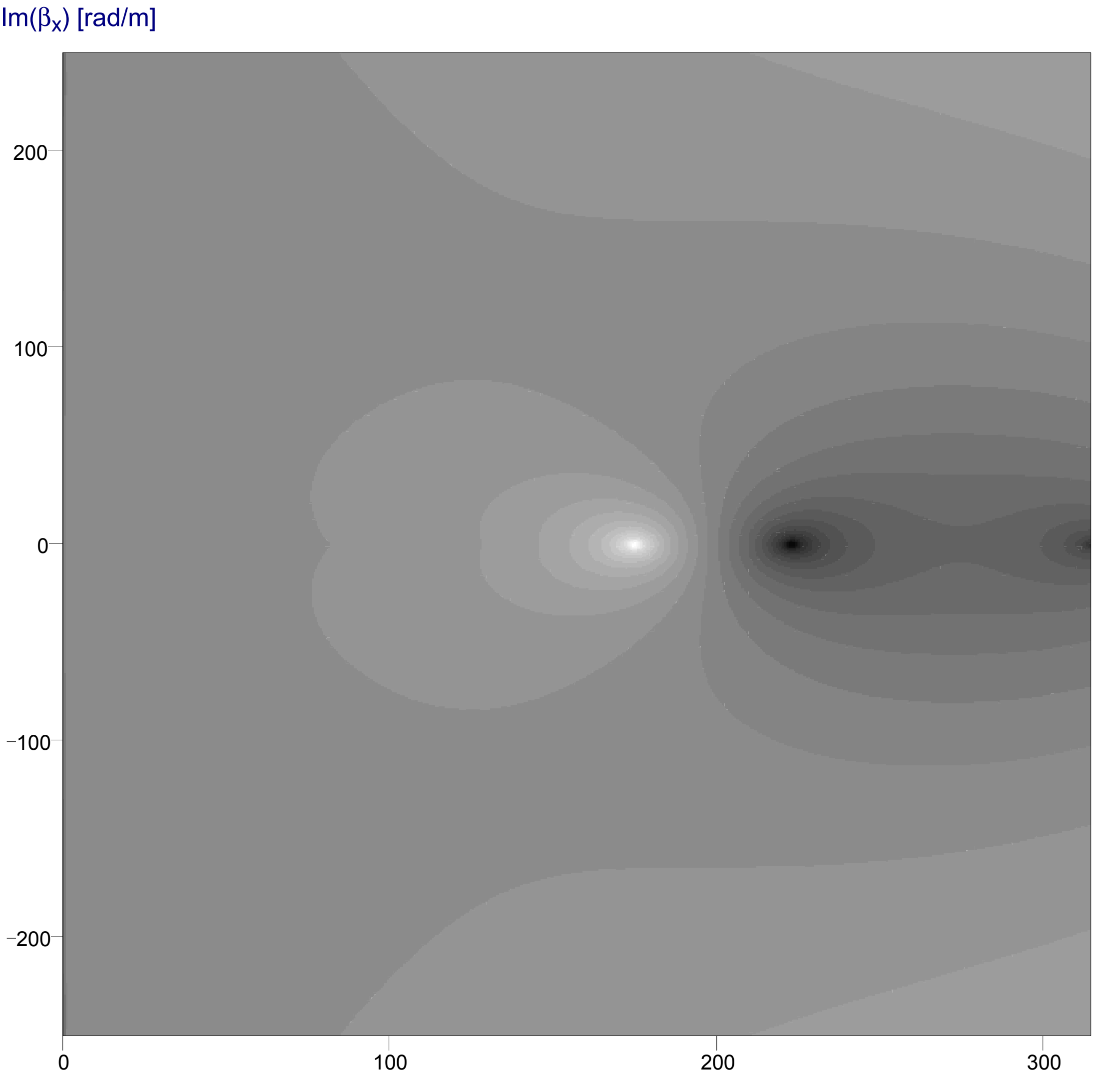
$s_{z1} := -j \cdot js_{z1}(\beta_x)$ $\quad$ $s_{z1} = -0.085 - 235.258j \cdot \frac{rad}{m}$

$s_{z3} := -j \cdot js_{z3}(\beta_x)$ $\quad$ $s_{z3} = -0.085 - 235.258j \cdot \frac{rad}{m}$

$N := 301 \qquad Start_x := 0 \cdot \frac{rad}{m} \qquad End_x := Re(k_2) \qquad Start_y := -250 \cdot \frac{rad}{m} \qquad End_y := 250 \cdot \frac{rad}{m}$

$x := 0, 1 .. N \qquad y := 0, 1 .. N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$

$B_{EP_{x,y}} := \log\left[\left| F_{EP}\left[\left(Start_x + x \cdot \Delta x \right) + \left[j \cdot \left(Start_y + y \cdot \Delta y \right) \right] \right] \right| \cdot m^2 \right]$



B_{EP}

H-type proper wave modes:

$$js_{z1}(\beta_x) := \frac{\mathrm{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right|}\cdot\sqrt{\beta_x^2 - k_1^2}$$

$$js_{z3}(\beta_x) := \frac{\mathrm{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)}{\left|\mathrm{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right|}\cdot\sqrt{\beta_x^2 - k_3^2}$$

$$F_{HP1}(\beta_x) := \left(-\frac{\mu_2}{\mu_1}\cdot\frac{\mu_2}{\mu_3}\cdot js_{z1}(\beta_x)\cdot js_{z3}(\beta_x) + k_2^2 - \beta_x^2\right)$$

$$F_{HP2}(\beta_x) := \tan\left(h\cdot\sqrt{k_2^2 - \beta_x^2}\right)$$

$$F_{HP3}(\beta_x) := \sqrt{k_2^2 - \beta_x^2}\cdot\left(\frac{\mu_2}{\mu_1}\cdot js_{z1}(\beta_x) + \frac{\mu_2}{\mu_3}\cdot js_{z3}(\beta_x)\right)$$

$$F_{HP}(\beta_x) := F_{HP1}(\beta_x)\cdot F_{HP2}(\beta_x) - F_{HP3}(\beta_x)$$

$\beta_x := k_2$ $\qquad \beta_x := \mathrm{root}\left(F_{HP}(\beta_x), \beta_x\right)$ $\qquad \beta_x = 315.075 - 0.063j \cdot \frac{\mathrm{rad}}{\mathrm{m}}$

$F_{HP}(\beta_x) = 0\cdot \mathrm{m}^{-2}$

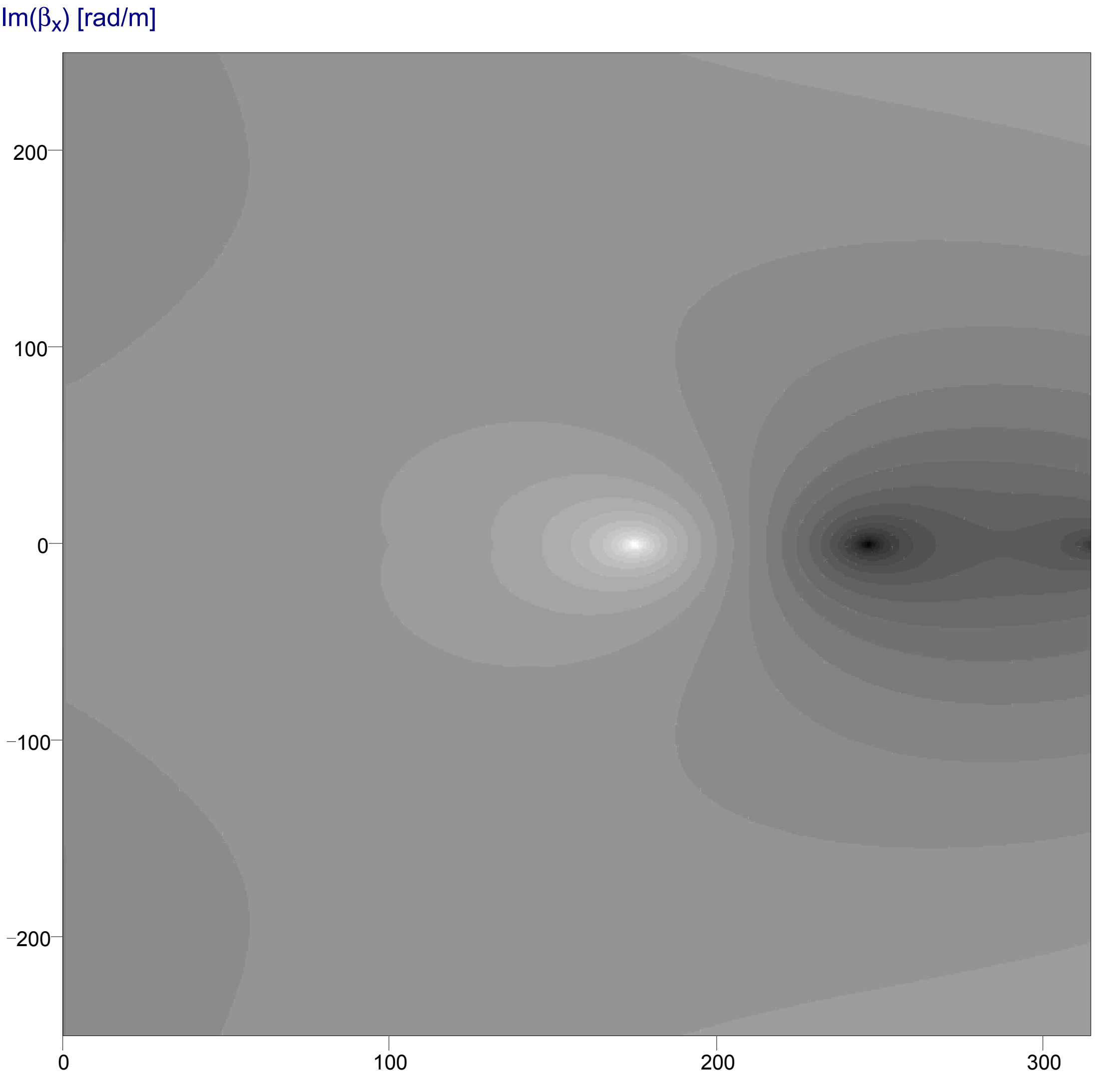
$s_{z1} := -j \cdot js_{z1}(\beta_x)$ $\qquad s_{z1} = -0.085 - 235.258j \cdot \frac{\mathrm{rad}}{\mathrm{m}}$

$s_{z3} := -j \cdot js_{z3}(\beta_x)$ $\qquad s_{z3} = -0.085 - 235.258j \cdot \frac{\mathrm{rad}}{\mathrm{m}}$

$N := 301 \qquad Start_x := 0 \cdot \frac{rad}{m} \qquad End_x := Re(k_2) \qquad Start_y := -250 \cdot \frac{rad}{m} \qquad End_y := 250 \cdot \frac{rad}{m}$

$x := 0, 1 .. N \qquad y := 0, 1 .. N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$

$B_{HP_{x,y}} := \log\left[\left|F_{HP}\left[\left(Start_x + x \cdot \Delta x\right) + \left[j \cdot \left(Start_y + y \cdot \Delta y\right)\right]\right]\right| \cdot m^2\right]$



B_{HP}

3.6 Plane Surface Waves along the Plane Interface of Two Half Spaces

Dispersion equations for the proper traveling waves along the plane interface of two half spaces (Fig. 3.1) can be obtained by letting h equal zero in the dispersion equations of the three layer case ((83) and (100)).

For E-type proper waves this results in

$$0 = \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right] \cdot \frac{\sqrt{\beta_x^2 - k_1^2}}{\sigma_1 + j\omega\varepsilon_1} + \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right] \cdot \frac{\sqrt{\beta_x^2 - k_3^2}}{\sigma_3 + j\omega\varepsilon_3}. \qquad (101)$$

The dispersion for H-type proper waves is

$$0 = \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_1^2}\right)\right] \cdot \frac{\sqrt{\beta_x^2 - k_1^2}}{\mu_1} + \operatorname{sign}\left[\operatorname{Re}\left(\sqrt{\beta_x^2 - k_3^2}\right)\right] \cdot \frac{\sqrt{\beta_x^2 - k_3^2}}{\mu_3}. \qquad (102)$$

EXAMPLE
The only propagating surface wave mode is a fast wave. Compare also the location of the null in this example with the anomalous null in Example 9 of Section 3.4.10.

Example: Plane Proper Waves along Two Half Spaces

Constants:

$c_0 := 299792458 \cdot \frac{m}{sec}$ $\mu_0 := 4 \cdot \pi \cdot 10^{-7} \cdot \frac{henry}{m}$ $\varepsilon_0 := \frac{1}{c_0^2 \cdot \mu_0}$ $\varepsilon_0 = 8.854 \cdot 10^{-12} \cdot \frac{farad}{m}$

Enter the material parameters:

$\sigma_1 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r1} := 2.26 - 0.5j$ $\mu_{r1} := 1 - 0j$

$\sigma_2 := 0 \cdot \frac{siemens}{m}$ $\varepsilon_{r2} := 1 - 0j$ $\mu_{r2} := 1 - 0j$

Enter the frequency:

$f := 10 \cdot 10^9 \cdot Hz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 6.283 \cdot 10^{10} \cdot Hz$ $\lambda_0 := \frac{c_0}{f}$ $\lambda_0 = 0.030 \cdot m$

Complex wave numbers:

$\varepsilon_1 := \varepsilon_{r1} \cdot \varepsilon_0$ $\varepsilon_2 := \varepsilon_{r2} \cdot \varepsilon_0$

$\mu_1 := \mu_{r1} \cdot \mu_0$ $\mu_2 := \mu_{r2} \cdot \mu_0$

$k_0 := \omega \sqrt{\varepsilon_0 \cdot \mu_0}$ $k_0 = 209.585 \cdot \frac{rad}{m}$

$k_1 := \sqrt{-j \cdot \omega \mu_1 \cdot \left(\sigma_1 + j \cdot \omega \varepsilon_1\right)}$ $k_1 = 316.974 - 34.645j \cdot \frac{rad}{m}$

$k_2 := \sqrt{-j \cdot \omega \mu_2 \cdot \left(\sigma_2 + j \cdot \omega \varepsilon_2\right)}$ $k_2 = 209.585 \cdot \frac{rad}{m}$

E-type proper wave modes:

$$F_{EP}(\beta_x) := \frac{Re\left(\sqrt{\beta_x^2 - k_1^2}\right)}{\left|Re\left(\sqrt{\beta_x^2 - k_1^2}\right)\right|} \cdot \frac{\sqrt{\beta_x^2 - k_1^2}}{\sigma_1 + j \cdot \omega \varepsilon_1} + \frac{Re\left(\sqrt{\beta_x^2 - k_2^2}\right)}{\left|Re\left(\sqrt{\beta_x^2 - k_2^2}\right)\right|} \cdot \frac{\sqrt{\beta_x^2 - k_2^2}}{\sigma_2 + j \cdot \omega \varepsilon_2}$$

$$\beta_x := \frac{k_1 + k_2}{2} \qquad \beta_x := \mathrm{root}\left(F_{EP}(\beta_x), \beta_x\right) \qquad \beta_x = 175.483 - 5.753j \cdot \frac{rad}{m}$$

$$F_{EP}(\beta_x) = -2.349 \cdot 10^{-4} - 2.742 \cdot 10^{-5} j \quad \cdot kg \cdot m^2 \cdot sec^{-1} \cdot coul^{-2}$$

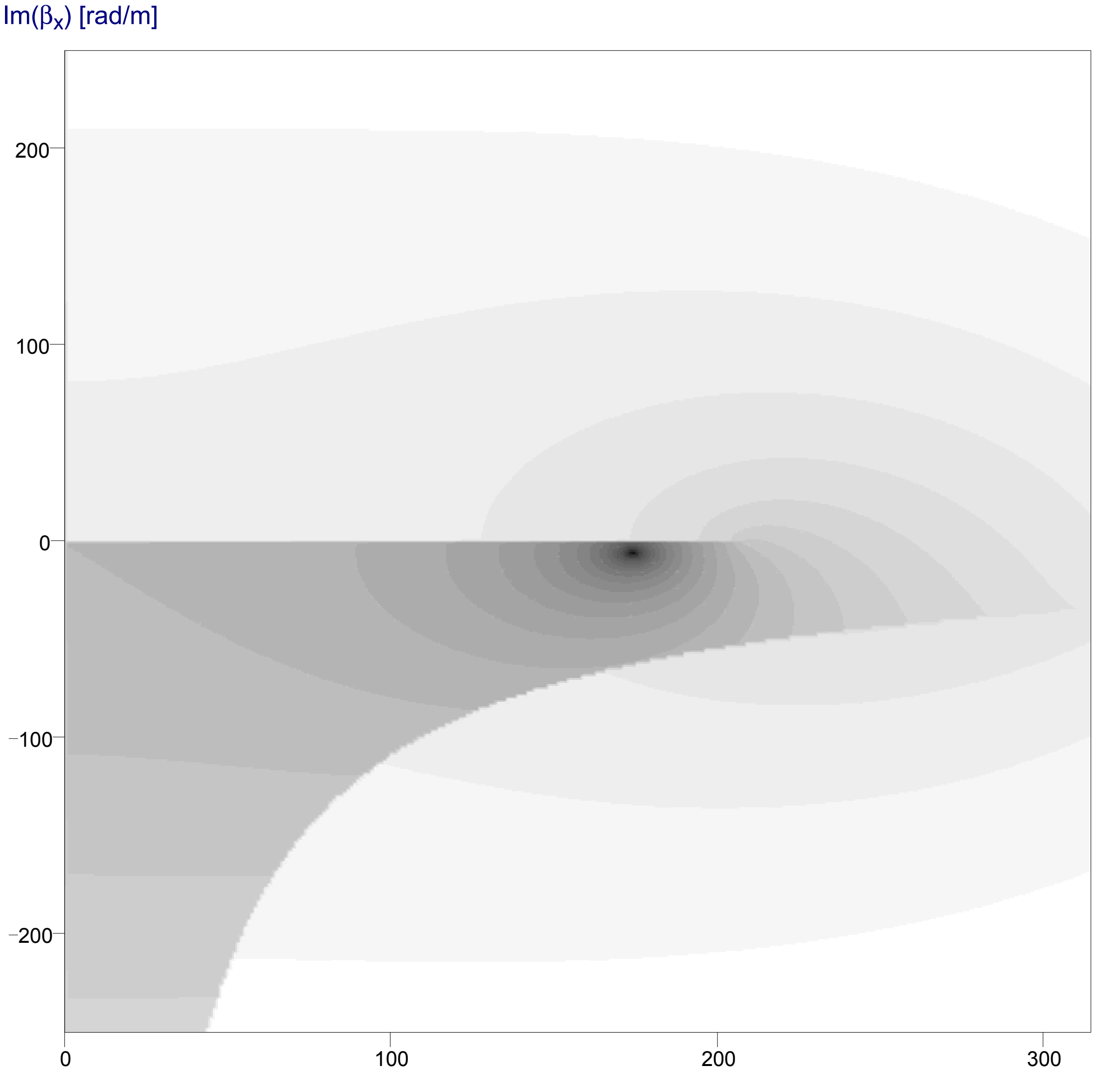
$$s_{z1} := -j \cdot \frac{Re\left(\sqrt{\beta_x^2 - k_1^2}\right)}{\left|Re\left(\sqrt{\beta_x^2 - k_1^2}\right)\right|} \cdot \sqrt{\beta_x^2 - k_1^2} \qquad s_{z1} = 264.448 - 37.708j \cdot \frac{rad}{m}$$

$$s_{z2} := -j \cdot \frac{Re\left(\sqrt{\beta_x^2 - k_2^2}\right)}{\left|Re\left(\sqrt{\beta_x^2 - k_2^2}\right)\right|} \cdot \sqrt{\beta_x^2 - k_2^2} \qquad s_{z2} = -115.072 - 8.773j \cdot \frac{rad}{m}$$

$N := 301 \qquad Start_x := 0 \cdot \frac{rad}{m} \qquad End_x := Re(k_1) \qquad Start_y := -250 \cdot \frac{rad}{m} \qquad End_y := 250 \cdot \frac{rad}{m}$

$x := 0, 1 .. N \qquad y := 0, 1 .. N \qquad \Delta x := \frac{End_x - Start_x}{N} \qquad \Delta y := \frac{End_y - Start_y}{N}$

$$B_{EP_{x,y}} := \log\left[\left| F_{EP}\left[\left(Start_x + x \cdot \Delta x \right) + \left[j \cdot \left(Start_y + y \cdot \Delta y \right) \right] \right] \right| \cdot \frac{siemens}{m} \cdot \frac{m}{rad} \right]$$



B_{EP}

3.7 Appendix A: The Phase Velocity of an Inhomogeneous Wave in a Loss Free Medium

The phase velocity of a wave in its propagation direction is given by

$$v_p = \frac{\omega}{\mathrm{Re}(\beta)}.$$

It will be shown now that *every inhomogeneous plane wave in a loss free medium* ($\mathrm{Im}(k) = 0$) *is a slow wave* (*i.e.* $\beta > k_0$) *in its direction of propagation.*

Assume that the inhomogeneous plane wave propagates in the xz-plane. The wave is then characterized by its wave vector

$$\vec{k} = k_x \vec{e}_x + k_z \vec{e}_z \qquad \text{(A1)}$$

where $k_x = \beta_x - j\alpha_x$ and $k_z = \beta_z - j\alpha_z$.

The direction of $\vec{k}$ corresponds to the propagation direction $\vec{\beta}$.
When the surrounding medium is loss free: $\mathrm{Im}(k) = 0$.
Hence, substituting the definitions of k_x and k_z into (A1) and squaring both sides results in [8, p. 166]

$$k^2 = \beta_x^2 + \beta_z^2 - \left(\alpha_x^2 + \alpha_z^2\right) \text{ because } -2j\left(\alpha_{2x}\beta_{2x} + \alpha_{2z}\beta_{2z}\right) = 0$$

$$\Rightarrow \vec{k}^2 = \vec{\beta}_2 - \vec{\alpha}_2$$

$$\Rightarrow \beta = \left\|\vec{\beta}\right\| = \sqrt{k^2 + \alpha^2} = k\sqrt{1 + \left(\frac{\alpha}{k}\right)^2} \geq k_0. \qquad \text{(A2)}$$

From this may be concluded that all inhomogeneous plane waves in a loss free medium are slow waves in their direction of propagation.

3.8 Appendix B: Proof of -j√x = √-x

Theorem

Let x be a complex number. Then

$$-j\sqrt{x} = \sqrt{-x}\,. \qquad \text{(B1)}$$

Proof

The above theorem can easily be proven by applying de Moivre's theorem to each side of (B1).

First, let θ be the argument of x.

Then, for the left hand side

$$\begin{aligned} -j\sqrt{x} &= -j\sqrt{|x|}\left[\cos\left(\frac{\theta + k360^\circ}{2}\right) + j\sin\left(\frac{\theta + k360^\circ}{2}\right)\right] \\ &= \sqrt{|x|}\left[\sin\left(\frac{\theta}{2} + k180^\circ\right) - j\cos\left(\frac{\theta}{2} + k180^\circ\right)\right]. \qquad \text{(B2)} \end{aligned}$$

The right hand gives

$$\begin{aligned} \sqrt{-x} &= \sqrt{|x|}\left[\cos\left(\frac{\theta + 180^\circ + k360^\circ}{2}\right) + j\sin\left(\frac{\theta + 180^\circ + k360^\circ}{2}\right)\right] \\ &= \sqrt{|x|}\left[\cos\left(\frac{\theta}{2} + 90^\circ + k180^\circ\right) + j\sin\left(\frac{\theta}{2} + 90^\circ + k180^\circ\right)\right] \\ &= \sqrt{|x|}\left[\sin\left(\frac{\theta}{2} + k180^\circ\right) - j\cos\left(\frac{\theta}{2} + k180^\circ\right)\right]. \qquad \text{(B3)} \end{aligned}$$

(B2) and (B3) are identical, therefore

$$-j\sqrt{x} = \sqrt{-x}\,.$$

3.9 Conclusions

A plane surface wave is defined as a plane wave that propagates along a plane interface of two different media without radiation.

Dispersion equations are derived for three different kinds of isotropic planar surface wave guiding structures. This is done by treating these structures as boundary-value problems and then solving these using Hertz potentials. The dispersion equations have a discrete number of both proper and improper solutions.

A distinction is made between E-type and H-type surface waves.

The concept of surface impedance has also been introduced. It was shown that E-type surface waves can only propagate along inductive surface impedances. H-type surface waves only propagate when the surface impedance is capacitive. *This is perhaps the important conclusion of this of chapter because it implies that isotropic surface wave absorbers are effective for one polarization only.*

The proper discrete eigenvalue spectrum does not form a complete set of eigenvalues along which the field of an open guiding structure may be expanded. However, the combination of the proper discrete eigenvalue spectrum and the continuous eigenvalue spectrum does.

Surface waves were also compared with other kinds of traveling waves. Some of these other waves are improper waves. By this is meant that they violate the radiation condition.

All traveling wave types can be either fast or slow waves. To prove this, the discrete eigenvalue spectrum is mapped onto the w-plane. This w-plane also prooves to be an excellent tool for designing surface wave guiding structures with specific propagation properties.

3.10 References

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