

THE DESIGN OF LOG-PERIODIC DIPOLE ANTENNAS

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SUMMARY

A mathematical analysis of the logarithmically periodic dipole class of frequency independent antennas, which takes into account the mutual coupling between dipole elements, is described. A high speed digital computer (ILLIAC) is used to calculate the input impedance, gain, and bandwidth, as well as the input current and voltage of the several elements. The results from this mathematical model are shown to be in good agreement with measurements. A step by step procedure, which enables one to design a log-periodic dipole antenna over a wide range of the parameters of input impedance, bandwidth, gain, and antenna size is presented.

1. INTRODUCTION

The log-periodic dipole is a linearly polarized frequency independent antenna of moderate gain. By frequency independent, we mean that the observable characteristics of the antenna such as the pattern and input impedance vary negligibly over a band of frequencies within the design limits of the antenna, and that this band may be made arbitrarily wide merely by properly extending the geometry of the antenna structure. The band limits of a given design are determined by nonelectrical restrictions; size governs the low frequency limit and precision of construction governs the high frequency limit.

This paper applies the results of a mathematical analysis of the log-periodic dipole antenna¹ to a step by step procedure which enables one to design a log-periodic dipole antenna over a range of the parameters that control the input impedance, bandwidth, gain, and antenna size.

2. DESCRIPTION OF THE LOG-PERIODIC DIPOLE ANTENNA

The LPD antenna pictured in Figure 1 and described by Figure 2, consists of a plurality of parallel linear dipoles arranged side by side in a plane. The lengths of the dipole elements and the spacing between elements form a geometric progression with a common ratio $\tau < 1$. The spacing factor σ is defined as the distance in wavelengths between a half-wave dipole and its next smaller neighbor. α is the angle subtended by the axis of the antenna and the ends of the elements on one side. σ is related to τ and α by $\sigma = (1/4)(1-\tau)\cot \alpha$. The elements are energized from a balanced, constant impedance feeder, adjacent elements

being connected to the feeder in an alternating fashion. The antenna may be fed by a balanced line connected to the small element end. Alternatively, a coaxial line running through one of the feeders from back to front may be used, the center conductor being connected to the other feeder as shown in Figure 3. In the latter method, the antenna becomes its own balun. When the operating frequency is within the design limits, radiation is endfire in the direction of the shorter elements, and the feeder currents behind the longest element are negligible; hence, the termination has little effect.

3. FORMULATION OF THE PROBLEM

The LPD antenna was invented by Isbell² at the University of Illinois in 1958 as a direct result of pioneering work on log-periodic structures by DuHamel³. At that time it was speculated that since the LPD is made of conventional dipole elements, it may be amenable to analysis in terms of the known properties of dipoles.

The analysis will be divided into two parts. The unknown voltages and currents along the feeder constitute the interior part of the problem. Determining the unknown far fields of the dipole elements constitutes the exterior part of the problem. Since the transverse dimensions of the feeder are small compared to wavelength, its principal function is to guide and distribute the energy to the radiating elements. The feeder furnishes us with the circuit properties of the antenna: impedance, voltage, and current. Thus, circuit techniques can be used in the solution of the interior problem.

4. THE INTERIOR PROBLEM

Insofar as the interior problem is concerned, the connection of the dipole elements with the feeder is equivalent to the parallel connection of two N terminal-pair circuits. One circuit consists of the feeder with alternating, properly spaced taps which represent the terminals to which the elements are eventually attached. The feeder circuit is shown schematically in Figure 4b. Note the inclusion of a terminating impedance Z_T . Z_T is arbitrary and can be varied to test for end effect. The parameters which describe this circuit are Z_0 , the feeder characteristic impedance; β , the feeder propagation constant; σ , τ , and d_1 which determine the spacing; and N .

Consider the admittance matrix of the feeder circuit $\bar{Y}_L$ and its inverse $\bar{Z}_L = \bar{Y}_L^{-1}$,

$$\bar{Y}_L = \begin{bmatrix} y_{11} & y_{12} & 0 & \cdots & 0 \\ y_{21} & y_{22} & y_{23} & 0 & \cdots & 0 \\ 0 & y_{32} & y_{33} & y_{34} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 & y_{NN-1} & y_N \end{bmatrix} \quad (1)$$

The y_{ij} 's are trigonometric functions. It becomes evident why all the elements off the diagonal by two or more are zero when we consider that y_{ij} is the current in terminal i due to a unit voltage at j , all terminal-pairs other than j being shorted. The short circuit restricts current flow to sections of line adjacent to the terminals to which the voltage is applied, hence no voltage is induced in the remaining sections.

The fields of the dipole elements determine the driving point impedances of the elements which shunt the feeder. The circuit properties of the elements are described by an N terminal pair circuit as shown in Figure 4a. Let $\bar{Z}_A$ be the element admittance matrix with $\bar{Z}_A = \bar{Y}_A^{-1}$,

$$\bar{Y}_A = \begin{bmatrix} y_{11} & y_{12} & y_{13} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & y_{NN} \end{bmatrix} \quad (2)$$

The y_{ij} 's represent the element to element coupling, the range of which extends throughout the antenna. The y 's are complex functions of τ , σ , h , and h/a , the height to radius ratio of the dipole elements. The method used to determine the y 's depends on the assumption of a sinusoidal current distribution on the dipoles.

The following relations obtain:

$$\bar{I}_L = \bar{Y}_L \bar{V}_L, \quad (3)$$

where $\bar{I}_L$ and $\bar{V}_L$ are column vectors which represent the N driving currents and response voltages respectively of the feeder circuit. Also,

$$\bar{I}_A = \bar{Y}_A \bar{V}_A, \quad (4)$$

where $\bar{I}_A$ and $\bar{V}_A$ are column vectors which represent the N driving currents and response voltages of the element circuit. If the corresponding terminals of the feeder and element circuits are connected in parallel, a new circuit is obtained as shown in Figure 4c. The response voltage vector is equal to either $\bar{V}_A$ or $\bar{V}_L$ since they are equivalent. The driving current vector of the new circuit is now the sum of

$\bar{I}_A$ and $\bar{I}_L$ due to conservation of current at a node. If equations (3) and (4) are added,

$$\bar{I} = \bar{I}_A + \bar{I}_L = \bar{Y}_A \bar{V}_A + \bar{Y}_L \bar{V}_L \quad (5)$$

$\bar{V}_L$ is set equal to $\bar{V}_A$ and factored,

$$\bar{I} = (\bar{Y}_A + \bar{Y}_L) \bar{V}_A. \quad (6)$$

We are interested in $\bar{I}_A$, the base current to the antenna terminals. Therefore,

$$\bar{I} = (\bar{Y}_A + \bar{Y}_L) \bar{Z}_A \bar{I}_A \quad (7)$$

or

$$\bar{I} = (\bar{U} + \bar{Y}_L \bar{Z}_A) \bar{I}_A, \quad (8)$$

where $\bar{U}$ is the unit matrix. The elements of $\bar{I}$ represent the input currents to the new circuit of Figure 4b. However, in the actual antenna all these currents are zero except the current at the feed point which is the driving current of the antenna. We may set the driving current equal to one ampere. Therefore,

$$\bar{I} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (9)$$

Equation (8) must be solved for $\bar{I}_A$. This can be done by inverting the matrix $\bar{T}$,

$$\bar{T} = \bar{U} + \bar{Y}_L \bar{Z}_A, \quad (10)$$

or by solving directly the set of simultaneous equations (8). The latter is easier to do in terms of computer time and accuracy due to the simplicity of $\bar{I}$. Once $\bar{I}_A$ is determined, $\bar{V}_A$ can be found by

$$\bar{V}_A = \bar{Z}_A \bar{I}_A \quad (11)$$

Note that the N^{th} element of $\bar{V}_A$ is the voltage across the smallest dipole; it is also the input impedance of the entire antenna since we assumed one ampere of input current.

5. THE EXTERIOR PROBLEM

The far field radiation pattern can be calculated once the element base currents $\bar{I}_A$ are known. The magnitude of the H-plane pattern is given by

$$|P_H(\phi)| = \left| \sum_{n=1}^N \frac{i_{An} (1 - \cos Bh_n)}{\sin Bh_n} \right|$$

$$\exp(-j\beta |x_n| \cos \varphi) \quad (12)$$

The magnitude of the E-plane pattern is given by

$$|P_E(\theta, \varphi)| = \left| \sin \theta \sum_{n=1}^N \frac{A_n [\cos(\beta h_n \cos \theta) - \cos \beta h_n]}{\sin \beta h_n} \right| \quad (13)$$

$$\exp(-j\beta |x_n| \sin \theta \cos \varphi) \quad (14)$$

The coordinate system used is shown in Figure 5. x_n is the distance from the origin to the n^{th} element.

6. THE COMPUTER SOLUTION

The above formulas were programed in complex number arithmetic for use of the ILLIAC, a high speed digital computer operated by the University of Illinois Graduate College. Because of the large amount of intermediate results which must be stored in the fast access electrostatic memory (capacity 1024 forty bit words), the program was split into six different parts, each stored on the slow access magnetic drum (capacity 16,384 forty bit words). They are:

1. Input: The descriptive parameters of the antenna are read into ILLIAC.
2. Computation of $\bar{Z}_A$
3. Computation of $\bar{Y}_L$
4. Matrix multiplication of $\bar{Y}_L \bar{Z}_A$ and solution of $\bar{I} = \bar{T} \bar{I}_A$.
5. Output $\bar{I}_A$, multiply $\bar{Z}_A \bar{I}_A = \bar{V}_A$ and output $\bar{V}_A$
6. Pattern calculation and scope display of patterns.

A control program calls each section into play as needed.

The input and basic output of the computer is by perforated paper tape which is translated by a teletypewriter. The patterns are calculated point by point and are plotted by the ILLIAC on a scope to which is attached an automatic 35 mm camera. The camera takes a picture of the completed pattern and advances the film into position for a new exposure. The details of the programing are not of general interest and will be omitted.

7. RESULTS OF COMPUTATIONS AND MEASUREMENTS

A. The Transmission Region

The mechanism of the transfer of energy from the feeder to the radiated field leads one to consider two fields on the structure.

One field originates at the feed point and propagates along the feeder in the direction of the larger elements. This is called the transmission field. The other field originates in the vicinity of the half-wavelength element and propagates to and beyond the tip of the antenna, manifesting itself in the radiated field. Separating the fields makes it possible to neglect the radiated field in the solution of the interior problem.

As the energy is launched from the feed point onto the small element end of the antenna, a TEM type wave is set up, supported by the feeders and the small elements which load the feeder log-periodically. This transmission line mode is evidenced by the feeder voltages $\bar{V}_L^+$. ($\bar{V}_L^+$ is the true transmission line voltage. $\bar{V}_L$ differs from $\bar{V}_L^+$ by an additional 180° phase at every other element due to the alternating feeder connection). A plot of the measured magnitude and phase of the feeder voltage is shown as a function of normalized distance from the apex in Figure 6. The calculated voltage $\bar{V}_L^+$ at the location of each element is also plotted. The results are in good agreement. The plot shows a steadily decreasing voltage away from the feed point to the short circuit which terminates the feeder beyond the largest element. The phase variation is essentially linear with distance up to the half-wavelength element. The slope of this curve is inversely proportional to the relative velocity of propagation v_t/c where v_t is the phase velocity of the transmission field and c is the velocity of light in the medium. This ratio is a function of the feeder characteristic impedance; as shown in Appendix A, it decreases as Z increases. The slow wave nature of the transmission field has been verified on a different type of LP structure by Bell, Elfving, and Franks.

B. The Active Region

The element base currents $\bar{I}_A$ in the region of half-wavelength elements (the "active region") provide a connection between the transmission field and the radiation field. The driving point impedances of the elements in the active region become predominately real, thus energy is efficiently coupled from the feeder to the radiating elements. The element base currents in the active region rise to a peak somewhat ahead of the resonant element, and then drop off rapidly as shown in Figure 7. As frequency is changed, the shape of this curve remains unchanged. That is, the active region moves along the antenna but its distance in wavelengths from the apex remains constant. The active region becomes deformed as it begins to include the front or back element on the antenna. When this happens, the upper or lower frequency limit is reached.

If the active region had no width, the operating bandwidth B would be given by the ratio of the length of the largest to smallest elements on the antenna. This ratio is called the bandwidth of the structure B_s .

$$B_s = \frac{l}{\tau} = \tau^{1-N} \quad (14)$$

Since the active region has some width, it is apparent that the operating bandwidth B is always less than B_s by a factor B_{ar} which is called the bandwidth of the active region. Thus,

$$B = B_s / B_{ar} \quad (15)$$

Placing numerical values on B and B_{ar} is complicated by the fact that the active region is not easily quantified. It has been found that the impedance and patterns remain essentially independent of frequency if the bandwidth of the active region is determined from the amplitude curve of Figure 7. Let R^+ and R^- be the distance to the apex of the -10 db points ($R^+ > R^-$). Then

$$B_{ar} = R^+ / R^- \quad (16)$$

A graph of B_{ar} vs α for several values of τ is given in Figure 8.

The phase of the current from element to element in the active region is shown in Figure 7. This is the phase which has to be considered when computing the radiation pattern. The slope of the curve in the active region implies a slow wave traveling towards the apex with a velocity of 0.36 c. This wave corresponds to the first backward space harmonic of a periodic structure made up of cells identical to the central cell of the active region. Mayes, Deschamps, and Patton⁵ have explained the operation of unidirectional frequency independent antennas in terms of backward wave radiation.

C. Radiation Patterns

An example of the computed patterns is shown in Figure 9. f_j denotes the frequency,

$$f_j = f_1 \tau^{1-j} \quad (17)$$

where f_1 is the frequency at which dipole number one is a half-wavelength long. Frequency independent behavior is observed over the design range of the antenna, with pattern break-up occurring at f_3 , beyond the high frequency limit. Measured and computed patterns are shown in Figure 10.

The principal pattern parameters are τ and σ . The directivity of the computed patterns was estimated from the E- and H-plane half-power beam widths by the approximate formula⁶

$$D \text{ in decibels} = 10 \log \frac{41253}{(BW_E)(BW_H)} \quad (18)$$

A map of constant directivity contours as a function of τ and σ is shown in Figure 11. In this figure Z_0 equals 100 ohms and h/a equals 125.

Let us define the operating region to be all values of τ and σ which result in single-lobed frequency independent patterns. The operating region is bounded on the right side of Figure 11 by values of τ so low that only one element on the antenna is near a half-wavelength long. In this case much of the energy in the transmission wave is passed beyond the active region and reflected at the end of the antenna, destroying the log-periodic performance. For values of σ less than .05 the directivity falls off rapidly and the input impedance varies with frequency. High values of τ on the left side of Figure 11 have not been completely investigated. In that case the length of the LPD antenna to produce a given bandwidth becomes excessive. The optimum σ indicated in Figure 11 is the value of σ for maximum directivity for a given τ . It is also the value of σ for which the input standing wave ratio is minimum, as explained in part D. For values of σ greater than the optimum, side lobes appear on the radiation pattern.

The directivity is independent of the characteristic impedance of the feeder for all practical values of Z_0 . The only effect other than the change in the input impedance is a shift in the location of the active region in the direction of smaller elements, as Z_0 is increased. The element height to radius ratio does affect the directivity; for each doubling of h/a the directivity decreases by about 0.2 db. in the range $50 < h/a < 10000$.

From an analysis of the phase of the far field radiation, the phase center of the LPD antenna has been found to lie within the active region. These results will be discussed in more detail in a forthcoming technical report⁷.

D. Input Impedance

A typical locus of calculated input impedance versus frequency on a Smith chart is shown in Figure 12. The points cluster around a mean resistance level R_0 . To determine R_0 , one draws a circle around the cluster. Its intersection with the resistance axis determines the minimum and maximum swing of resistance. R_0 is then given by

$$R_0 = \sqrt{R_{\max} R_{\min}} \quad (19)$$

The standing wave ratio with respect to R_0 is given by

$$SWR = \sqrt{\frac{R_{\max}}{R_{\min}}} \quad (20)$$

The mean resistance level R_o is determined principally by the characteristic impedance of the feeder Z_o , as shown in Figure 13. The dependence on Z_o is also given by an approximate formula for R_o which has been found to agree with all measured and computed results to date, within 10 per cent.

$$R_o = Z_o \sqrt{1 + \frac{Z_o}{4\sigma'Z_a}} \quad (21)$$

where

$$\sigma' = \frac{\sigma}{\sqrt{\tau}} \quad (22)$$

is a mean spacing factor and

$$Z_a = 120 (\ln \frac{h}{a} - 2.25) \quad (23)$$

is an average characteristic impedance of a short dipole as a function of h/a . The derivation of equation 21 is given in Appendix A. R_o as a function of τ and α is plotted in Figure 14 for $h/a = 125$, and $Z_o = 100$. It is seen that R_o decreases with increasing τ and increasing α .

The standing wave ratio with respect to R_o has a minimum value of about 1.1:1 at the optimum value of σ . As σ is decreased below the optimum the SWR rises above 1.8:1 at $\sigma = 0.05$. These values represent a lower bound; the measured SWR is usually somewhat greater, as shown in Figure 13.

8. DESIGN PROCEDURE

For most applications, we are interested in designing an antenna which has a given directivity and input impedance over a given frequency band. Once these electrical characteristics are specified, we have to decide the relative importance of reducing the number of elements or the size of the antenna. These two characteristics are not independent. The number of elements is determined mainly by τ ; as τ increases the number of elements increases. The antenna size is determined mainly by the boom length (the distance between the smallest and largest element), which depends primarily on σ . As σ increases the boom length increases.

With these facts in mind a preliminary choice of τ and σ can be made from the graph of Figure 11. To determine the boom length and the number of elements, the bandwidth of the structure B_s must be found. The dependent parameter α is given by

$$\tan \alpha = \frac{1 - \tau}{4\sigma} \quad (24)$$

or by the nomograph of Figure 17. Consult Figure 8 to determine the bandwidth B_{ar} for the given values of τ and α . The bandwidth of the structure B_s is then given by

$$B_s = B_{ar} \quad (25)$$

where B is the required operating bandwidth. The geometry of the LPD antenna now provides a relation between the boom length relative to the longest operating wavelength $\lambda_{\max}$ since the length of the first element is always made equal to $\frac{\lambda_{\max}}{2}$.

$$\frac{L}{\lambda_{\max}} = \frac{1}{4} \left(1 - \frac{1}{B_s}\right) \cot \alpha \quad (26)$$

where L is the boom length between the largest and smallest elements. A nomograph of equation 26 is given in Figure 18. The number of elements required is found from the equation

$$N = 1 + \frac{\log B_s}{\log 1/\tau} \quad (27)$$

a nomograph of which is given in Figure 19.

The design procedure for the principal LPD parameters is now complete. It is likely that the first estimate of τ and σ will lead to a longer boom length than necessary, so a revision must be made in τ and σ , and the above procedure should be repeated several times until the minimum boom length is found. On the other hand, the minimum boom length design may require an excessive number of elements or the SWR for that value of σ may be too high, so a re-adjustment of τ and σ might be called for.

Once the final values of τ and σ are found, the characteristic impedance of the feeder Z_o must be determined so as to give the required input impedance R_o . Structural considerations would probably dictate the element height to radius ratio h/a . The average characteristic impedance of the element, Z_a , can then be found from

$$Z_a = 120 (\ln h/a - 2.25) \quad (28)$$

or from the graph of Figure 15. Using the relative element impedance Z_a/R_o and $\sigma' = \sigma/\sqrt{\tau}$, and inverting formula 21 gives the characteristic impedance of the feeder relative to R_o .

$$\frac{Z_o}{R_o} = \frac{1}{8\sigma'Z_a/R_o} + \sqrt{\frac{1}{(8\sigma'Z_a/R_o)^2} + 1} \quad (29)$$

A graph of equation 29 is given in Figure 16. Once Z_o is found, the design is complete. In practice, the feeder termination Z_T is a short circuit a distance of $\lambda_{\max}/8$ or less behind element number one. In several cases a short circuit at the terminals of element number one yielded satisfactory results; in these cases the shorted element acted as a passive reflector at the lowest frequencies.

As an example of the design procedure, it

was desired to build an LPD antenna with a directivity of 9.5 db, a bandwidth of 1.75, an input impedance of 80 ohms, and an h/a of 118. Values of $\tau = 0.92$ and $\sigma = 0.12$ were selected from the graph of Figure 11. From Figure 17 α is 9.5° . The bandwidth of the active region B for $\alpha = 9.5^\circ$ is 1.46 from Figure 8. This means that the structure bandwidth B must be 2.55. The relative boom length $L/\lambda_{\max}^s$ is 0.91 and $N = 12$ from Figures 18 and 19. The height of element number one is $\lambda_{\max}/4$ and a short circuit terminates the feeder at a distance $0.185 \lambda_{\max}$ behind the first element. A feeder impedance of 100 ohms was used, as determined from Figures 15 and 16. The resulting antenna is pictured in Figure 1. The average measured directivity was 9.52 over a 1.8:1 band. The input impedance was 73 ohms over a 1.8:1 band, and the SWR with respect to 73 ohms was less than 1.3:1. Twelve other laboratory models agreed with the data in Figure 11 with an average error of 0.35 db, and a maximum error of 1.5 db. Thus we have the formulas needed to design a log-periodic dipole antenna. The nomographs are particularly useful, since they allow one to work out many preliminary designs without resorting to computations.

APPENDIX A. APPROXIMATE FORMULA FOR R_o

It is an established fact that the standing wave ratio with respect to R_o of a well made LPD is small, usually less than 1.4:1. The small standing wave ratio is observed no matter how severe the front truncation, as long as a well-formed active region remains on the antenna. These two facts suggest that the observed R_o is actually the characteristic impedance of a transmission line made up of the feeder and the small elements, and that the active region is a good match to this transmission line.

If we consider the capacitive loading of the small elements, we note that the capacity is proportional to the length of the elements and that the spacing d_j of element j is proportional to the length of element j . As an approximation, then, the capacity per unit length is constant. Thus we add to the nominal capacity per unit length of unloaded feeder a term which represents the capacitive loading of the small elements. Consider the approximate formula for the input impedance of a dipole antenna,

$$Z = -j Z_a \cot \beta h, \quad (30)$$

Z_a is an average characteristic impedance of the dipole.

$$Z_a = 120 \left(\ln \frac{h}{a} - 2.25 \right), \quad (31)$$

This is Jordan's⁸ modification of a formula from Siegel and Labus. If the dipole is small, β is the free space propagation constant. Replacing the cotangent function by its small argument approximation, the capacity of the n -th dipole is given by

$$C_n = \frac{h_n}{c Z_a}, \quad (32)$$

where c is the velocity of light in vacuo. If we use the mean spacing at dipole n ,

$$d_{\text{mean}} = \sqrt{\frac{d_n d_{n-1}}{\tau}} = \frac{d_n}{\sqrt{\tau}}, \quad (33)$$

the average capacity per unit length is given by

$$\Delta C = \frac{C_n}{\text{length}} = \frac{h_n \sqrt{\tau}}{c d_n Z_a}, \quad (34)$$

But h_n/d_n is related to the spacing factor σ by

$$\sigma = \frac{1}{4} \frac{d_n}{h_n}, \quad (35)$$

hence

$$\Delta C = \frac{\sqrt{\tau}}{4 c Z_a \sigma}. \quad (36)$$

Since

$$Z_o = \sqrt{\frac{L_o}{C_o}} \quad \text{and} \quad c = \frac{1}{\sqrt{L_o C_o}}, \quad (37)$$

$$R_o = \sqrt{\frac{L_o}{C_o + \Delta C}}. \quad (38)$$

Or

$$R_o = \frac{Z_o}{\sqrt{m}} \quad (39)$$

where

$$m = 1 + \frac{Z_o}{Z_a} \frac{\sqrt{\tau}}{4 \sigma}. \quad (40)$$

The new propagation constant β_L is given by

$$\beta_L = \beta_o \sqrt{m}. \quad (41)$$

Thus we have a formula for the mean resistance level R_o and the propagation constant in terms of the spacing parameter σ , τ and the h/a ratio.

Inverting the formula to find the feeder impedance Z_o in terms of R_o , we find

$$Z_o = R_o \left[\frac{1}{8 \sigma' \frac{Z_a}{R_o}} + \sqrt{\left(\frac{1}{8 \sigma' \frac{Z_a}{R_o}} \right)^2 + 1} \right], \quad (42)$$

where

$$\sigma' = \frac{\sigma}{\sqrt{\tau}}$$

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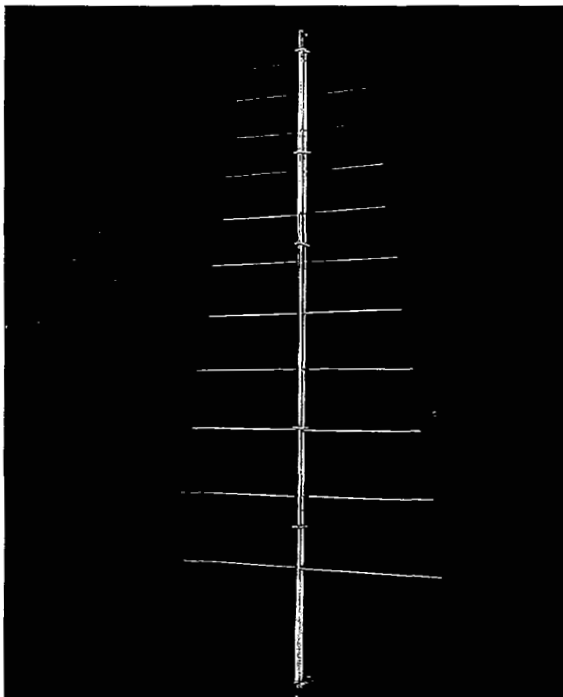


Fig. 1. A log-periodic dipole antenna. $\tau = 0.92$, $\sigma = 0.12$, $\alpha = 9.5^\circ$, $N = 12$, $Z_0 = 100 \Omega$, and $h/a = 118$.

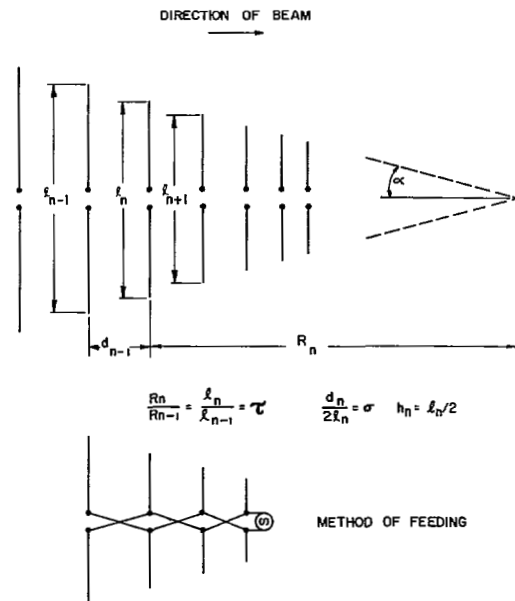


Fig. 2. The log-periodic dipole antenna, with definitions of parameters.

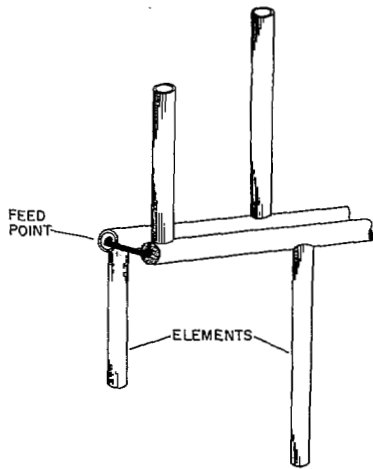


Fig. 3. Connection of elements to balanced feeder and feed point details.

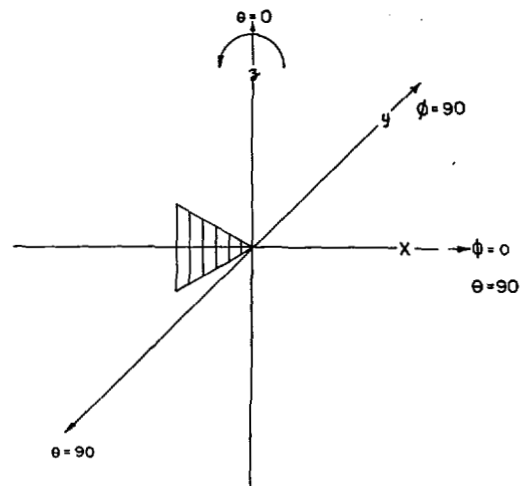


Fig. 5. LPD pattern coordinate system.

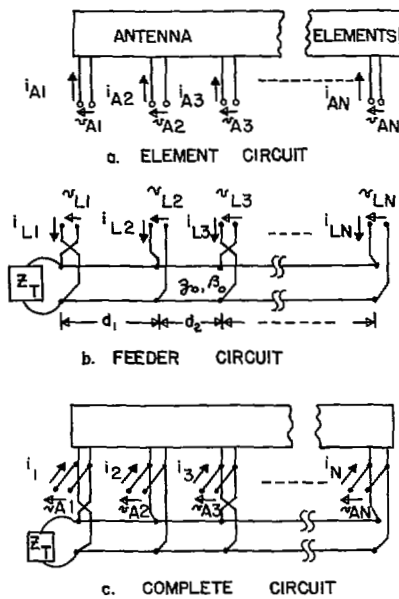


Fig. 4. Schematic circuits for the LPD interior problem.

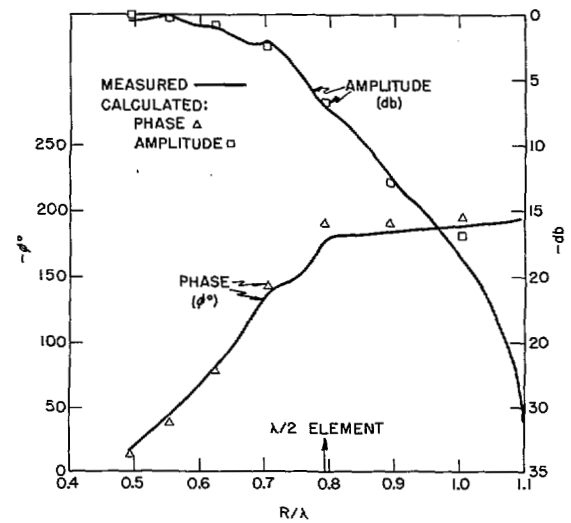


Fig. 6. Relative magnitude and phase of feeder voltage vs R/λ at f_3 , $\tau = 0.888$, $\sigma = 0.089$, $\alpha = 17.5^\circ$, $N = 8$, $Z_0 = 100 \Omega$, $h/a = 125$, short circuit termination 0.128λ behind element number one.

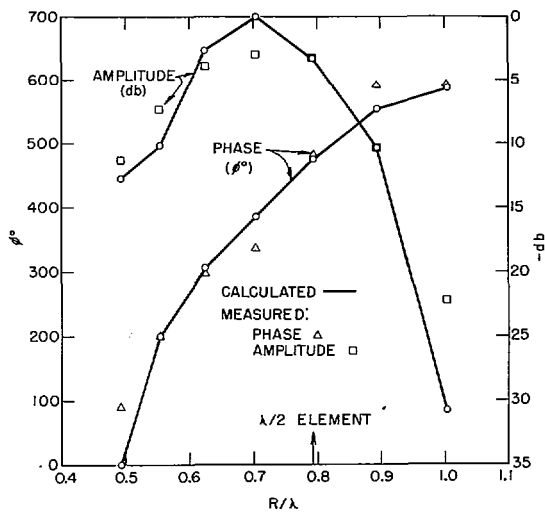


Fig. 7. Relative magnitude and phase of element base current vs R/λ at f_3 , $\tau = 0.888$, $\sigma = 0.089$, $\alpha = 17.5^\circ$, $N = 8$, $Z_0 = 100 \Omega$, $h/a = 125$, short circuit termination 0.128λ behind element number one.

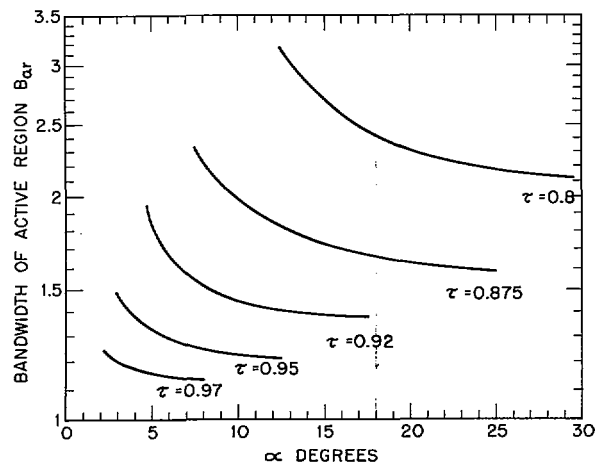


Fig. 8. Bandwidth of action region B_{ar} vs α for several values of τ , for $Z_0 = 100 \Omega$, $h/a = 125$.

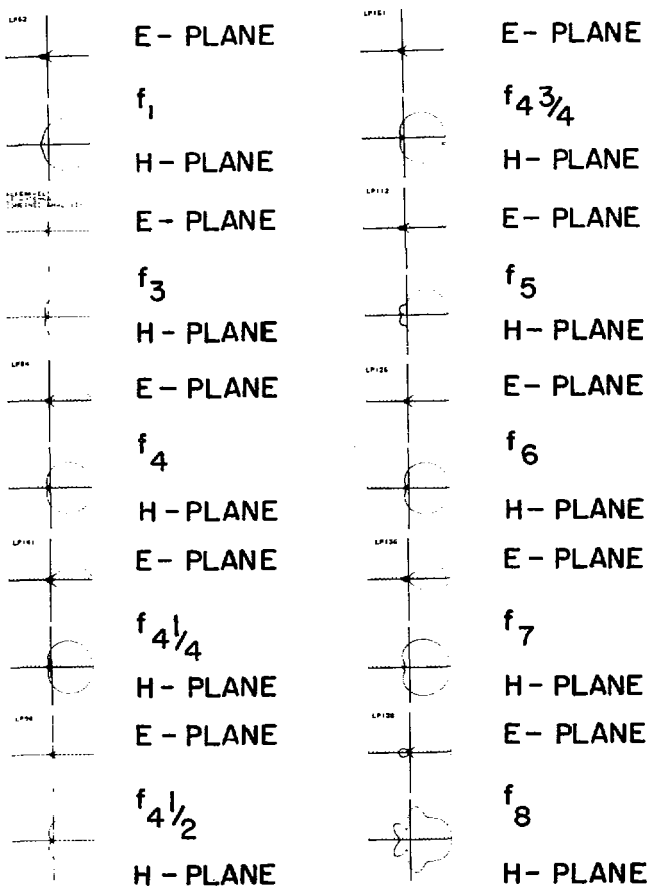


Fig. 9. Computed patterns vs frequency, $\tau = 0.888$, $\sigma = 0.089$, $\alpha = 17.5^\circ$, $Z_0 = 100 \Omega$, $h/a = 125$, Z_T = short at $0.1 \lambda_{max}$ behind element number one.

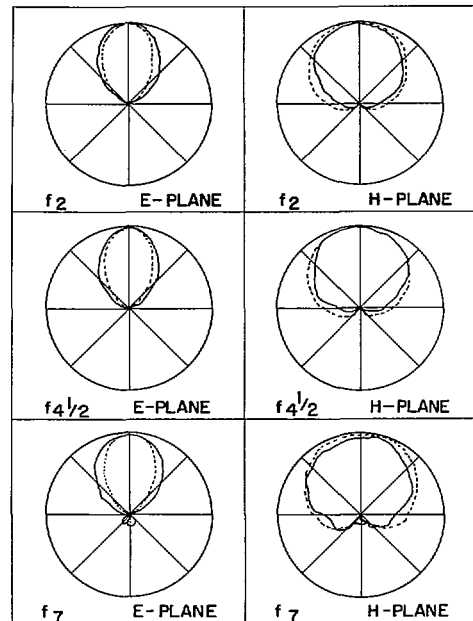


Fig. 10. Computed and measured patterns, $\tau = 0.888$, $\sigma = 0.089$, $\alpha = 17.5^\circ$, $Z_0 = 100 \Omega$, $h/a = 125$, Z_T = short at $0.1 \lambda_{max}$ behind element number one.

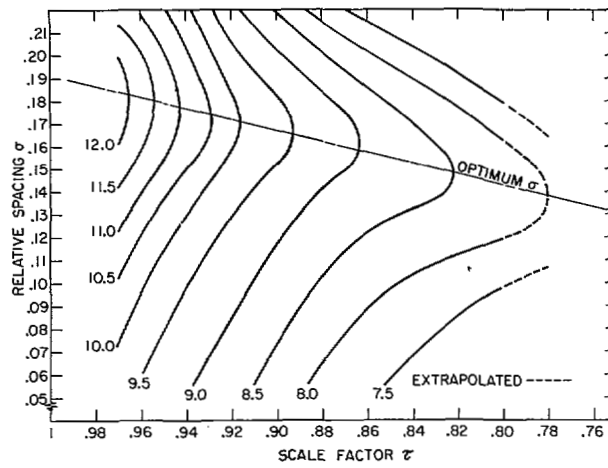


Fig. 11. Constant directivity contours in decibels vs τ and σ . $Z_0 = 100 \Omega$, $h/a = 125$, $Z_T = 100 \Omega$, optimum σ indicates maximum directivity for a given value of τ .

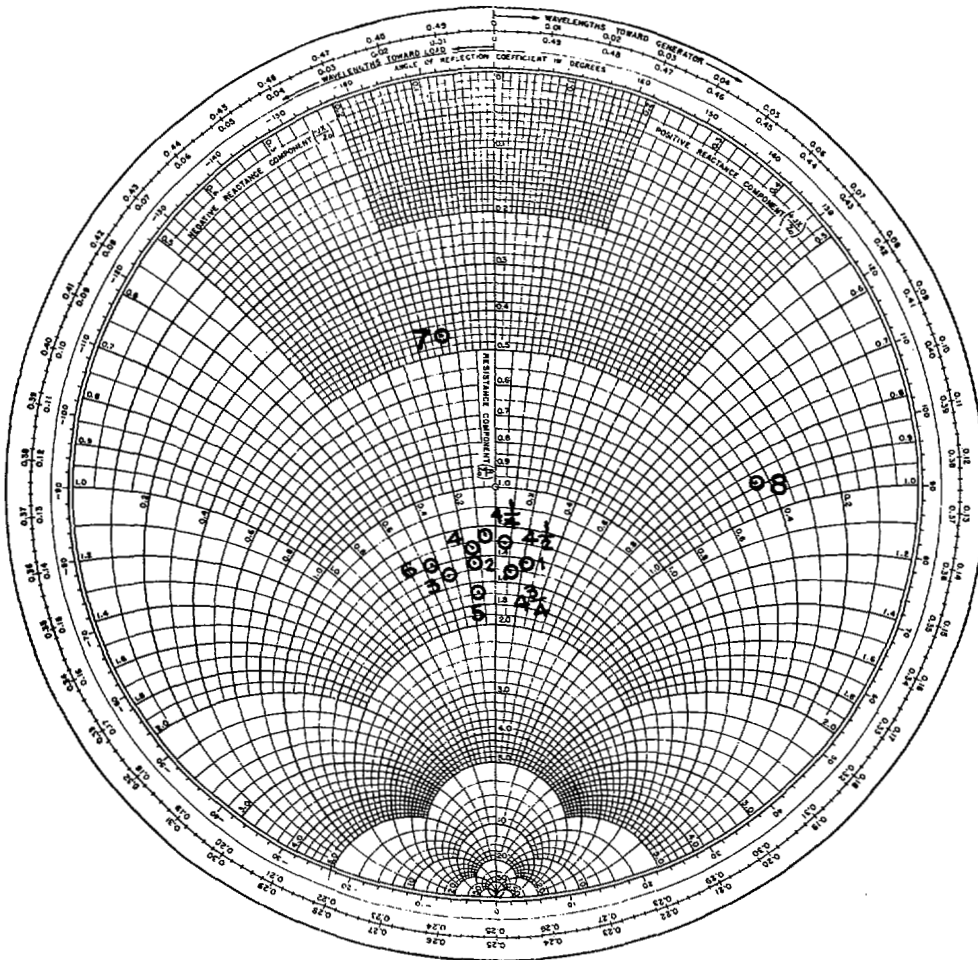


Fig. 12. Computed input impedance vs frequency, $\tau = 0.888$, $\sigma = 0.089$, $\alpha = 17.5^\circ$, $Z_0 = 100 \Omega$, $Z_T = 100 \Omega$, $h/a = 125$.

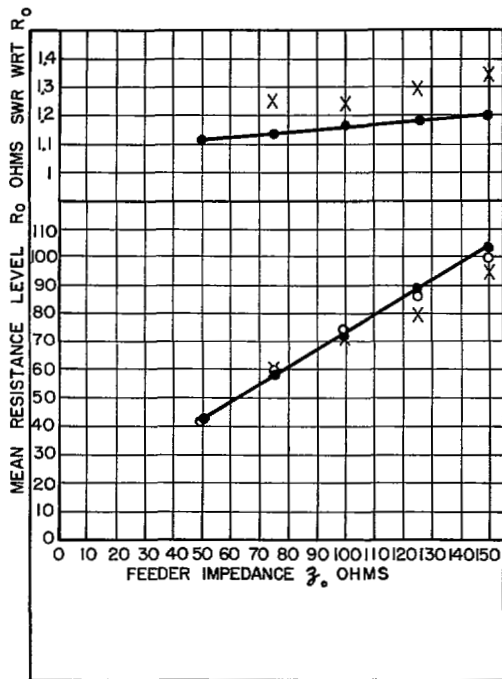


Fig. 13. Mean resistance level R_0 and SWR vs feeder impedance, $\tau = 0.888$, $\sigma = 0.089$, $\alpha = 17.5^\circ$, $h/a = 125$, $Z_T = Z_0$, —•—•— computed, xxx experimental, ooo approximate formula.

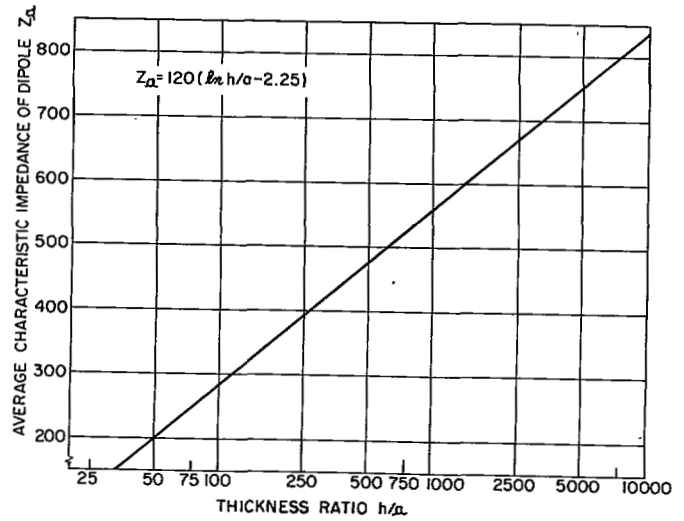


Fig. 15. Average characteristic impedance of a dipole, $Z_d = 120 (\ln h/a - 2.25)$.

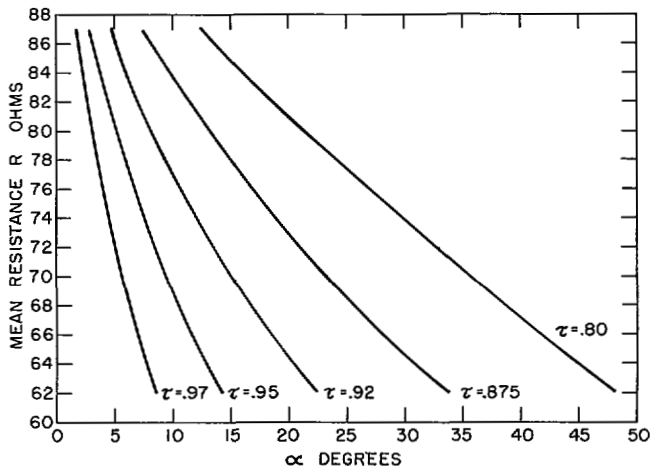


Fig. 14. Mean resistance level R_0 vs α and τ , $Z_0 = 100 \Omega$, $h/a = 125$, from approximate formula.

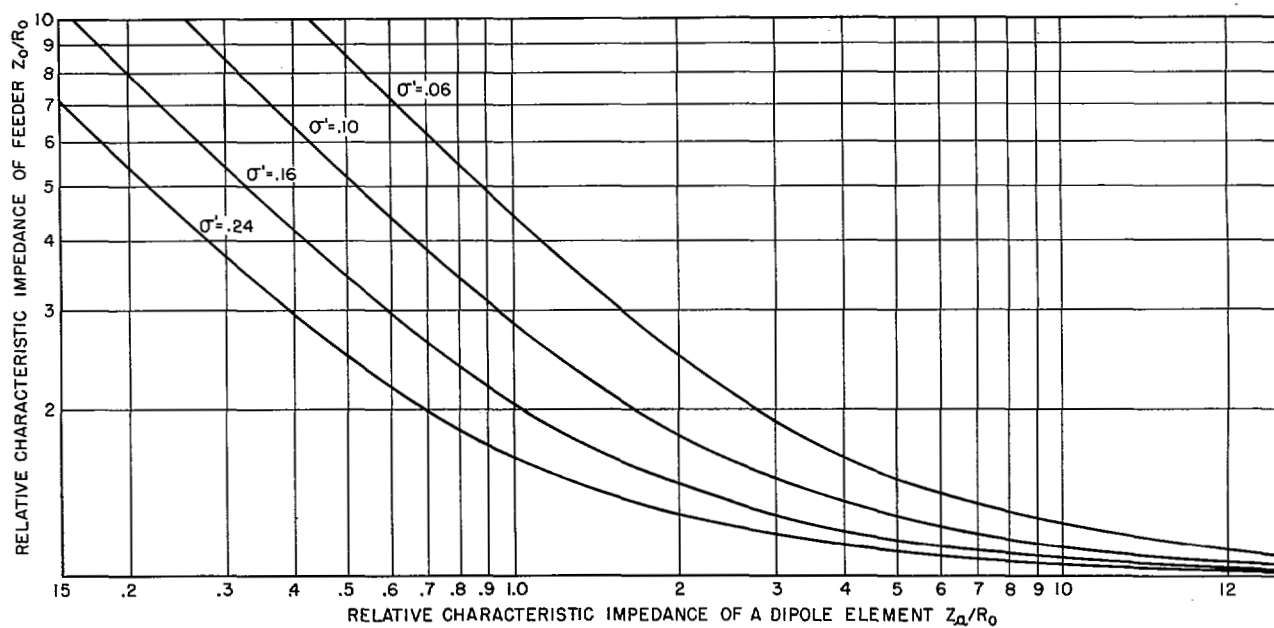


Fig. 16. Relative feeder impedance vs relative dipole impedance for several values of σ .

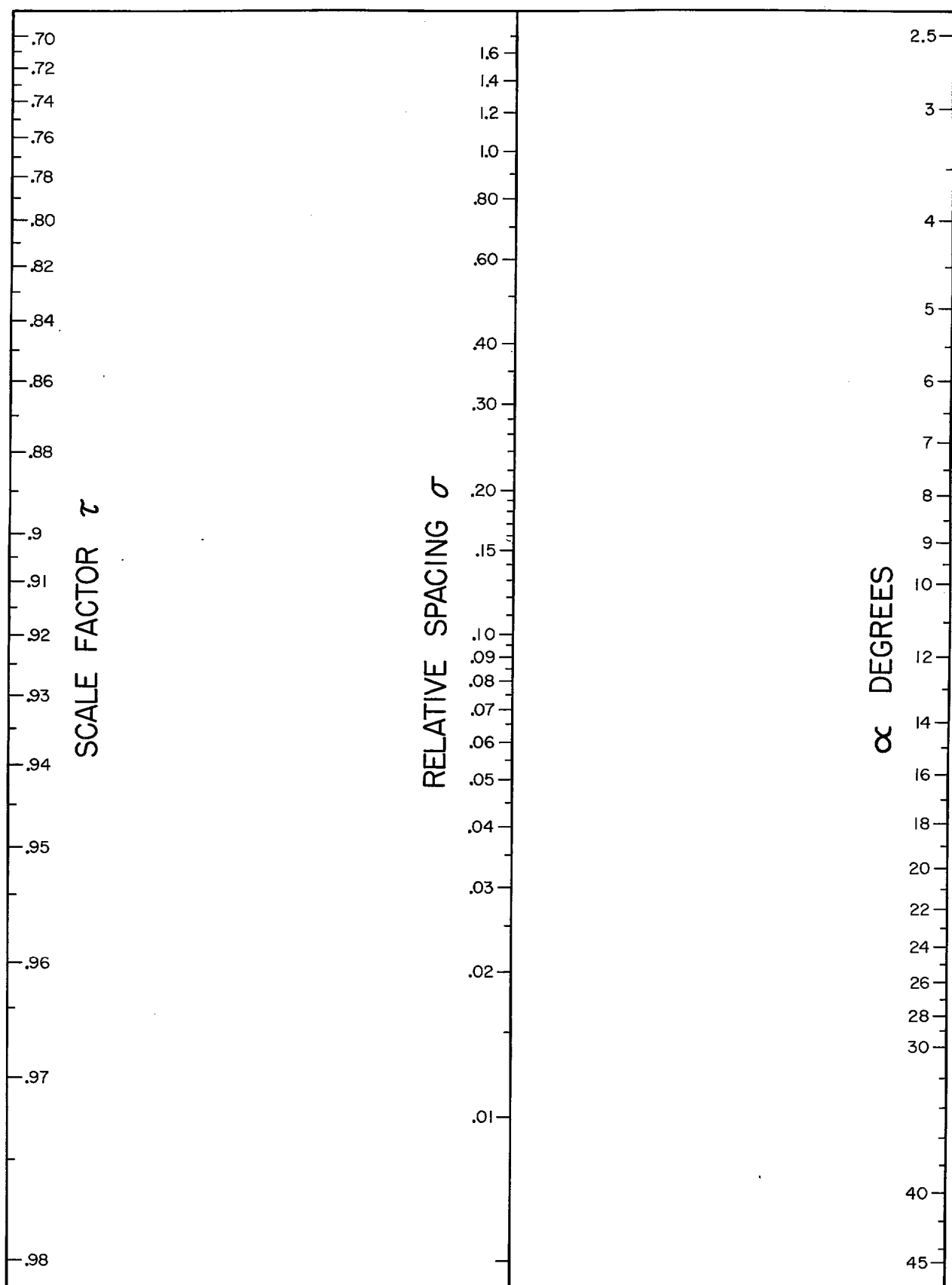


Fig. 17. Nomograph of $\sigma = \left(\frac{1}{4}\right)(1 - \tau) \cot \alpha$.

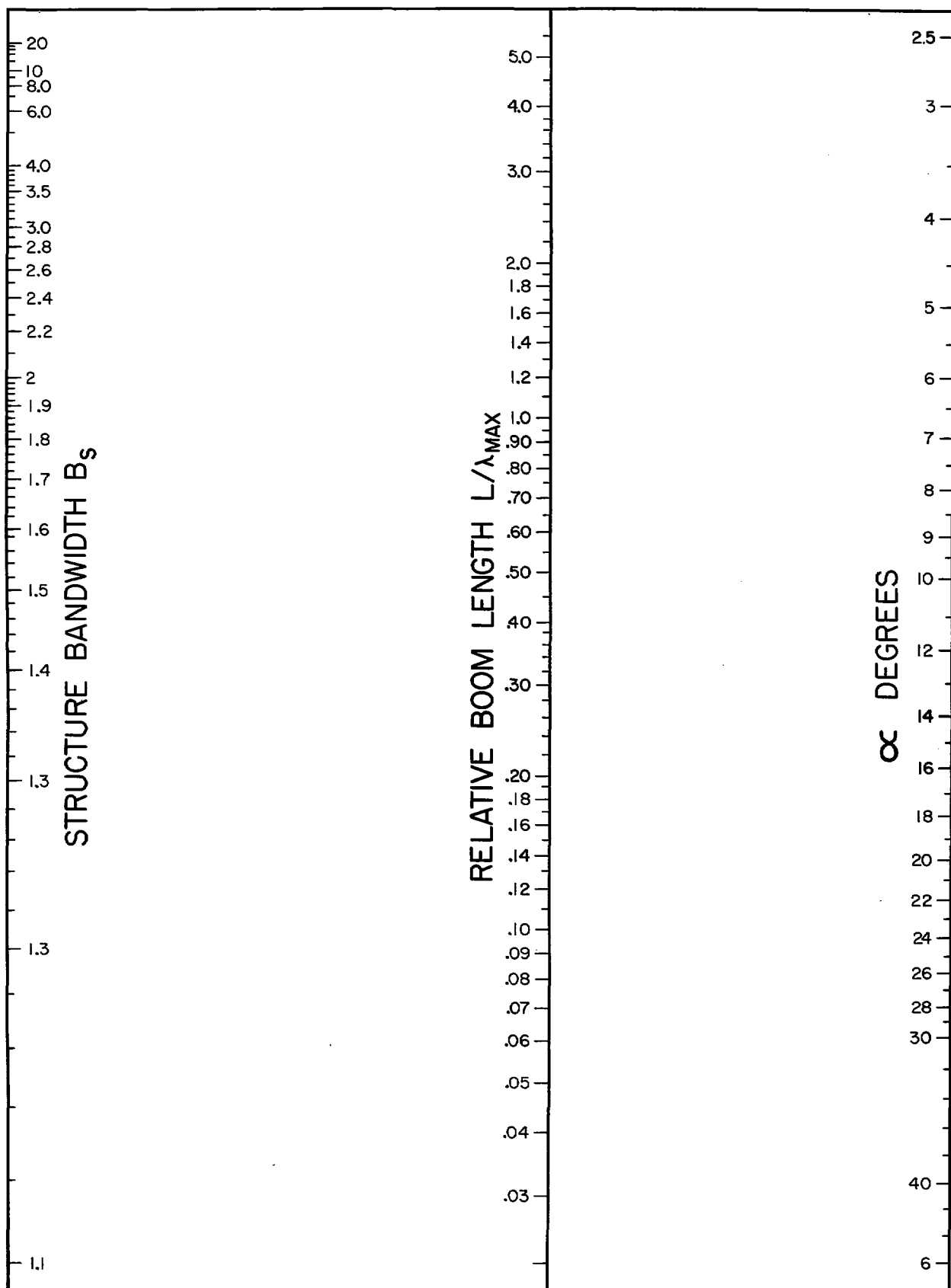


Fig. 18. Nomograph of $L/\lambda_{max} = \left(\frac{1}{4}\right) \left(1 - 1/B_S\right) \cot \alpha$.

